14.01

Microeconomics

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(84)

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Problem Set 2

Problem 1: **a)** At equilibrium,

$$
\begin{aligned}
Q_D &= Q_s \\
570 - 100P &= -30 + 200P \\
600 &= 300P \\
P &= 2
\end{aligned}
$$

So the equilibrium price is \$2 and the quantity demanded is $570 - 100P = 570 - 200 = 370$ million gallons

b) The elasticity of demand at the equilibrium price is given by

$$\epsilon = \frac{\frac{dQ}{dp}}{\frac{p}{Q}} = \frac{-100}{\frac{2}{370}} = \frac{-200}{370} = -0.54$$

c) If the fuel efficiency of cars increases, the demand for gasoline will decrease. The demand curve shifts left, and the equilibrium price and quantity decrease.

Problem 2: The income rises by $\frac{8}{40} = 0.2$, and $\xi = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta Y}{Y}} = -\frac{1}{2}$. So

$$\frac{\Delta Q}{Q} = -\frac{1}{2}\frac{1}{5} = -\frac{1}{10}$$

Thus,

$$\Delta Q = -\frac{1}{10}50 = -5$$

So they will purchase 45 oz of hamburgers.

Problem 3: **a)**

$$\epsilon = \frac{\frac{dQ}{dp}}{\frac{p}{Q}} = \frac{-5p^{-6}}{\frac{p}{p^{-5}}} = \frac{-5p^{-6}}{p^{-6}} = -5$$

The price elesticity of demand is constant with respect to price.

b)

$$\epsilon = \frac{\frac{dQ}{dp}}{\frac{p}{Q}} = \frac{-10p^{-6}}{\frac{p}{2p^{-5}}} = \frac{-10p^{-6}}{2p^{-6}} = -5$$

His elasticity of demand is also -5 at any price.

c) A $x\%$ change in price causes a $-5x\%$ change in quantity, so the total amount Ms. Smith spends on apples decreases as price increases.

d) The price elasticity of demand is

$$\epsilon = \frac{\frac{dQ}{dp}}{\frac{p}{Q}} = \frac{-b}{\frac{p}{a-bp}} = \frac{-ba + b^2 p}{p} = \frac{-ba}{p} + b^2$$

Problem 4: **a) True.** The elasticity of demand is a more negative number at higher prices. At high prices, the quantity demanded is low, so a relatively small increase in price corresponds to a relatively large decrease in quantity; at low prices, the inverse is true.

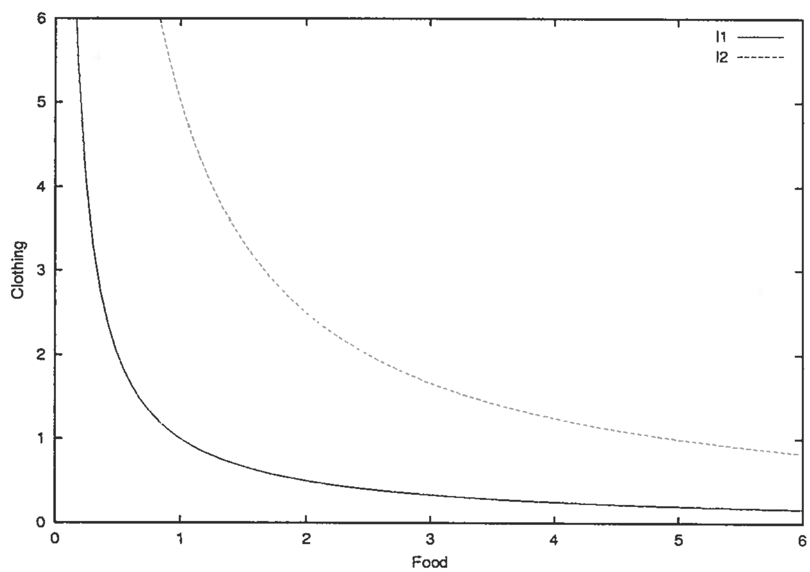
b) False. If there is excess demand, the price will be driven up by market forces until an equilibrium price is reached.

c) False. The price of computers might also fall due to a shift in the demand curve, if e.g. alternatives become cheaper.

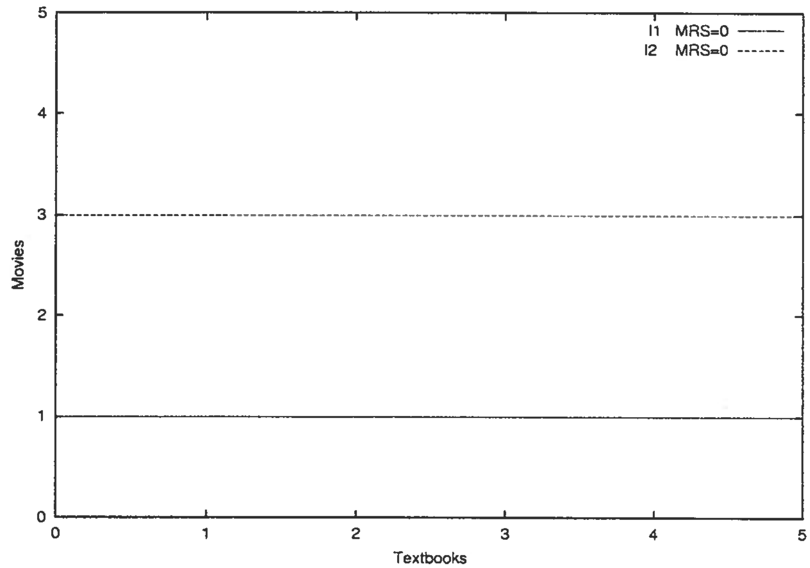
d) True. Utility is maximized at the highest indifference curve that intersects the budget line, which is where the budget line is tangent to the indifference curve, i.e. the marginal rate of substitution is equal to the ratio of the prices.

e) False. Doubling the prices can either double the revenue from late fees (if demand is perfectly inelastic), or decrease the number of late registrations (if demand is elastic), but not both.

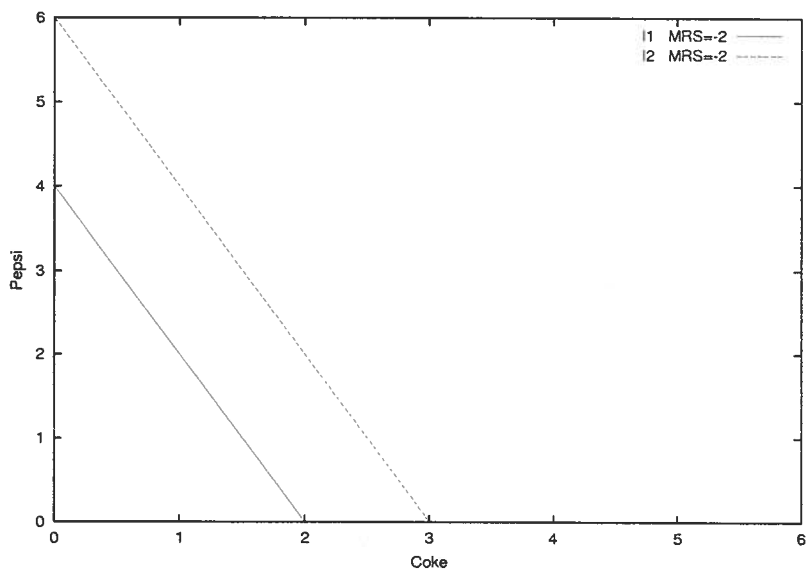
Problem 5: In each graph, I_2 is a higher indifference curve (increased utility) than I_1. **a)** A decreasing marginal rate of substitution corresponds to a convex demand curve. We have no other information.



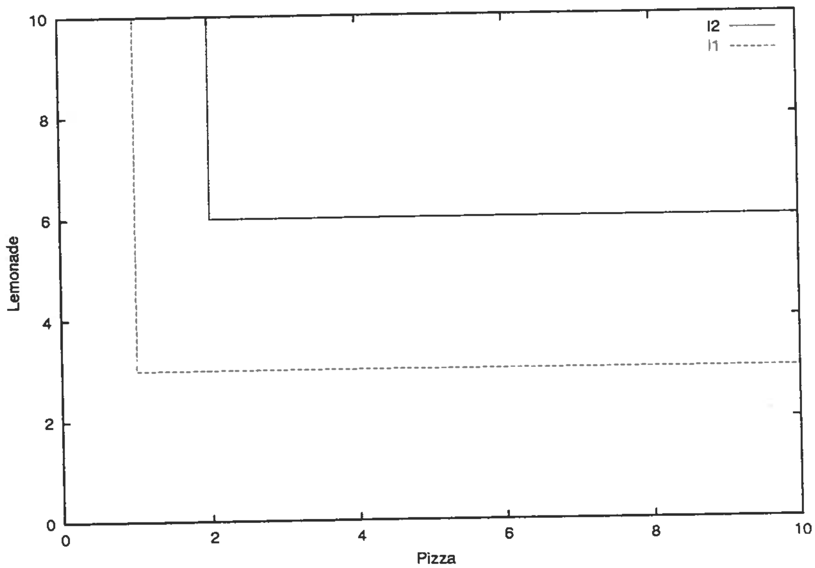
b) With movies on the x axis and textbooks on the y axis, the indifference curves will be vertical, since textbooks do not affect utility.



c) The indifference curves will be linear since Pepsi and Coke are perfect substitutes. $(2,2)$ and $(3,0)$ are on the same indifference curve, so the indifference curves have slope -2.



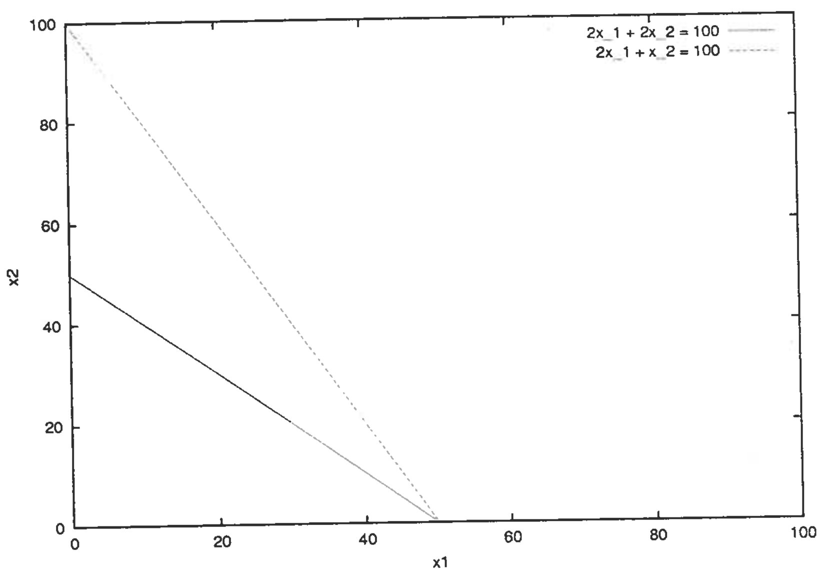
d) The indifference curves are L-shaped, with corner points along the line $y = 2x$. The MRS is zero in the horizontal segments and (negative) infinite in the vertical segments.



Problem 6: **a)** Twice as much of good 2 can be purchased. The budget constraint changes to

$$2x_1 + x_2 = 100$$

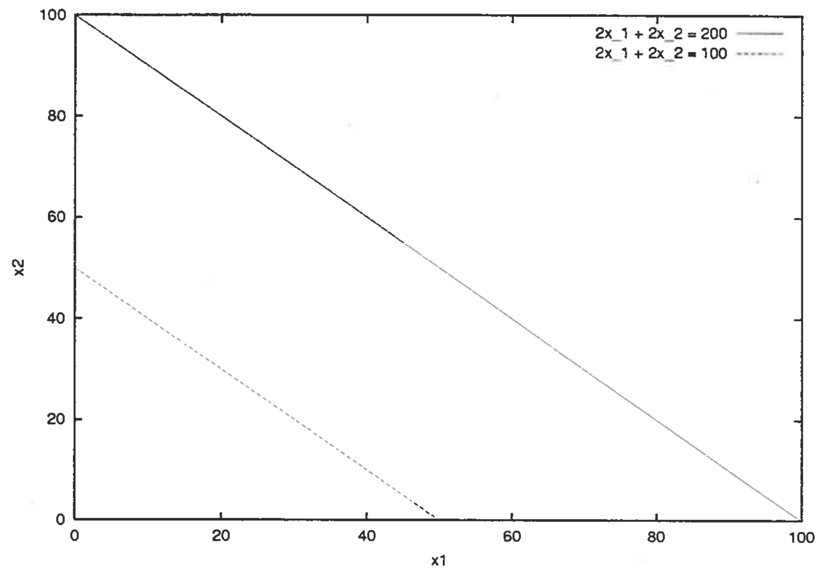
This is shown graphically below.



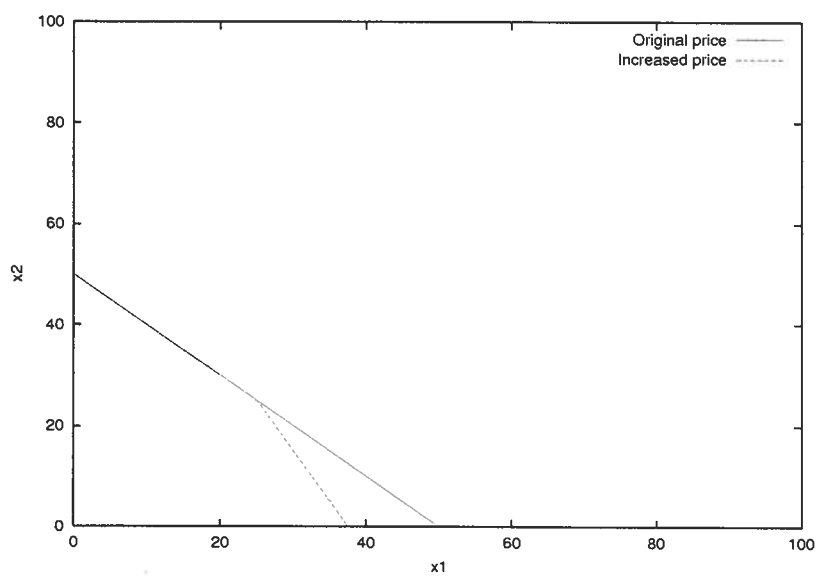
b) The budget constraint changes to

$$2x_1 + 2x_2 = 200$$

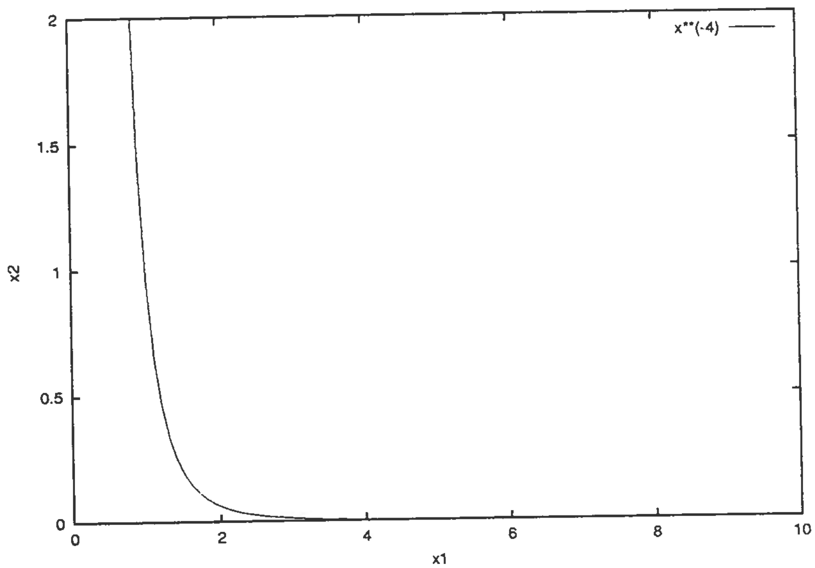
as shown below.



c) The price increase corresponds to the piecewise-defined function below.



Problem 7: **a)** Consider the indifference curve $U(x_1, x_2) = 1$, or $x_1^4 x_2 = 1$. It is plotted below.



b) The marginal utility of good 1 is

$$MU_1 = \frac{\partial U(x_1, x_2)}{\partial x_1} = 4x_1^3 x_2$$

and the marginal utility of good 2 is

$$MU_2 = \frac{\partial U(x_1, x_2)}{\partial x_2} = x_1^4$$

So the marginal rate of substitution is

$$MRS = -\frac{MU_2}{MU_1} = -\frac{x_1^4}{4x_1^3 x_2} = -\frac{x_1}{4x_2}$$

$$-\frac{MU_1}{MU_2}$$

c) Utility is maximized at the point where the marginal rate of substitution equals the marginal rate of transformation:

$$-\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}$$
$$MU_1 p_2 = MU_2 p_1$$
$$4x_1^3 x_2 p_2 = x_1^4 p_1$$
$$4x_2 p_2 = x_1 p_1$$

From the budget constraint, we have

$$p_1 x_1 + p_2 x_2 = I$$
$$x_2 = \frac{I - p_1 x_1}{p_2}$$

So

$$\begin{aligned}
4p_2\frac{I-p_1x_1}{p_2} &= x_1p_1 \\
4I - 4x_1p_1 &= x_1p_1 \\
x_1 &= \frac{4I}{5p_1}
\end{aligned}$$

And applying the budget constraint again

$$\begin{aligned}
\frac{4I}{5p_1}p_1 + p_2x_2 &= I \\
p_2x_2 &= I - \frac{4I}{5} \\
x_2 &= \frac{I}{5p_2}
\end{aligned}$$

Problem 8: The utility function is $U(x_1, x_2) = \min\{ax1, bx_2\}$. The indifference between the two commodity bundles tells us

$$\min\{50a, 10b\} = \min\{20a, 100b\}$$

Assume without loss of generality that $a = 1$. Then we must determine b.

Claim 1. *b is between $1/5$ and 5.*

Proof. By contradiction, assume $b > 5$. Then $\min\{50, 10b\} = 50$ and $\min\{20, 100b\} = 20$. But these two quantities are equal because the bundles have the same utility, which is a contradiction. Next suppose $b < 1/5$. Then $\min\{50, 10b\} = 10b$ and $\min\{20, 100b\} = 100b$. But this can only be true if b equals zero, in which case the utility function is invalid. So we have proven the claim. □

By the claim, we now know $\min\{50, 10b\} = 10b$ and $\min\{20, 100b\} = 20$. These are equal, so $b = 2$.

This allows us to compute the utility of the other two bundles:

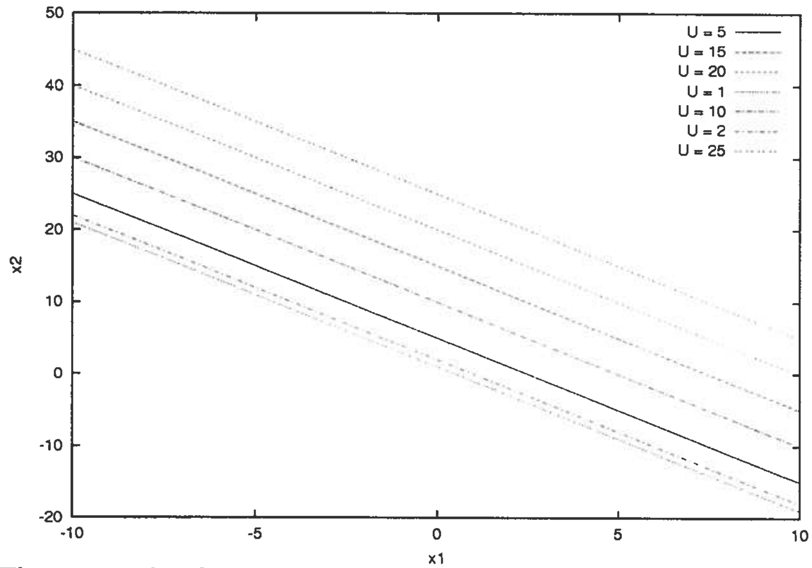
$$U(30, 10) = 20$$

and

$$U(20, 12) = 20$$

So the two bundles are equally ranked.

Problem 9: **a)**



The marginal utility of good 1 is

$$MU_1 = \frac{\partial U(x_1, x_2)}{\partial x_1} = 2$$

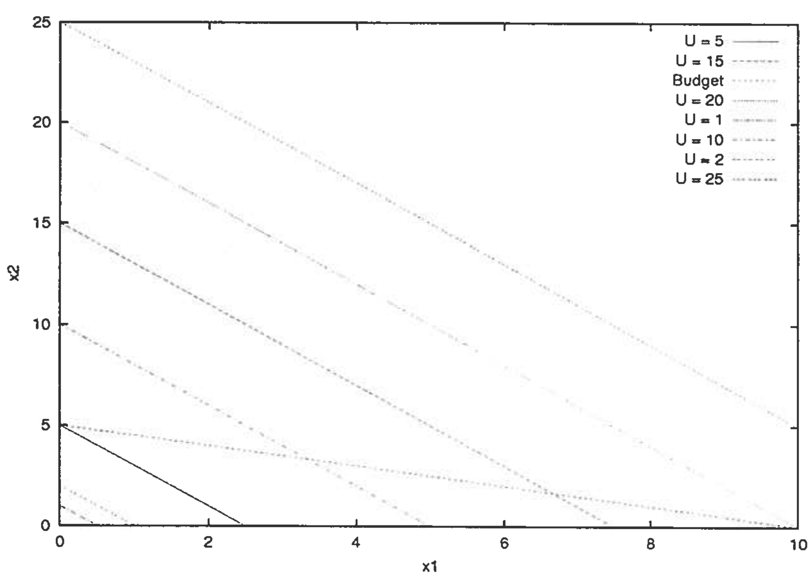
and the marginal utility of good 2 is

$$MU_2 = \frac{\partial U(x_1, x_2)}{\partial x_2} = 1$$

So the marginal rate of substitution is

$$MRS = -\frac{MU_2}{MU_1} = -\frac{1}{2}$$

b)



The budget line intersects the highest indifference curve at $(x1, x2) = (10, 0)$. At this point, $U = 20$. This is a corner solution, so the MRS need not equal the ratio of the prices.

c) The marginal rate of substitution is constant, so the goods are perfect substitutes: one unit of x_1 has equal utility as two units of x_2. So if $\frac{p_1}{p_2} > \frac{1}{2}$, then the entire budget will be spent on x_2, and if the ratio is less than 1/2, then the entire budget will be spent on x_1. If the ratio is exactly 1/2, then the two choices are equivalent in price and utility.

So

$$x_1 = \begin{cases} \frac{I}{p_1} & : \frac{p_1}{p_2} > \frac{1}{2} \\ 0 & : \frac{p_1}{p_2} < \frac{1}{2} \end{cases}$$

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$$x_2 = \begin{cases} 0 & : \frac{p_1}{p_2} > \frac{1}{2} \\ \frac{I}{p_2} & : \frac{p_1}{p_2} < \frac{1}{2} \end{cases}$$

Problem 10: The marginal utility of good 1 is

$$MU_1 = \frac{\partial U(x_1, x_2)}{\partial x_1} = \frac{2}{x_1}$$

and the marginal utility of good 2 is

$$MU_2 = \frac{\partial U(x_1, x_2)}{\partial x_2} = \frac{1}{x_2}$$

So the marginal rate of substitution is

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$$MRS = -\frac{MU_2}{MU_1} = -\frac{x_1}{2x_2}$$

At the bundle $(3, 2)$, the MRS is $-\frac{3}{4}$. This is the utility maximizing consumption bundle, so the MRS is equal to the ratio of the prices:

$$\frac{p_1}{p_2} = \frac{3}{4}$$

So we can find p_1:

$$p_1 = \frac{3}{4}p_2 = \frac{9}{2}$$

And at the utility maximizing consumption bundle, all of the income is spent on these goods, so

$$I = x_1p_1 + x_2p_2 = 3\frac{9}{2} + 4(6) = \frac{57}{2}$$