14.01
Microeconomics
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Problem Set 3

Problem 1:

a) The marginal utility of butter is 1 and the marginal utility of margarine is 5, so

$$MRS_{bm} = -\frac{MU_b}{MU_m} = -\frac{1}{5}$$

The indifference curves are parallel lines with slope of $-1/5$, with butter on the x axis and margarine on the y axis.

b) The two goods are perfect substitutes, with five sticks of butter equal to one stick of margarine, so at equal prices a consumer will only consume margarine.

$$M = \frac{\$45.00}{\$1.50} = 30$$
$$B = 0$$

c) The prices are still equal, so the consumer will spend his or her entire income on margarine.

$$M(I) = \frac{I}{\$1.50}$$
$$B(I) = 0$$

d) Let p be the ratio of the price of margarine to the price of butter: $p = \frac{p_M}{p_B}$. If p is less than 5, the entire income will be spent on margarine as this maximizes utility; if p is greater than 5, then the entire income will be spent on butter. If p equals exactly 5, then nothing can be said about the relative amounts of butter and margarine purchased except that the total cost is equal to the income I:

$$M(p, I) = \begin{cases} \frac{I}{p_M} & : p < 5 \\ 0 & : p > 5 \\ \frac{I - p_B B}{p_M} & : p = 5 \end{cases}$$

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$$B(p, I) = \begin{cases} 0 & : p < 5 \\ \frac{I}{p_B} & : p > 5 \\ \frac{I - p_M M}{p_B} & : p = 5 \end{cases}$$

Problem 2:

a) The two goods are perfect substitutes at a 1:1 equivalence ratio, and the price of movies is less.

$$m = \frac{\$200}{\$1} = 200$$
$$b = 0$$

b) The two goods are still perfect substitutes, but now books are cheaper.

$$m = 0$$
$$b = \frac{\$200}{\$0.50} = 400$$

c) To maximize utility, one unit of books must be purchased per unit of movies. So

$$m = b = \frac{\$200}{\$2 + \$1} = \frac{200}{3}$$

Problem 3:

a) The marginal utility of clothes is

$$MU_C = \frac{\partial U}{\partial C} = \frac{3}{4C}$$

and the marginal utility of food is

$$MU_F = \frac{\partial U}{\partial F} = \frac{1}{4F}$$

so the marginal rate of substitution is

$$MRS = -\frac{MU_C}{MU_F} = -\frac{12F}{4C} = \frac{3F}{C}$$

At the equilibrium point, the MRS is equal to the MRT, so

$$\frac{3F}{C} = \frac{p_C}{p_F} = \frac{20}{5}$$
$$C = \frac{3}{4}F$$

We apply the budget constraint $20C + 5F = 2000$ and find

$$(F, C) = (100, 75)$$

b) The cost of the same bundle is now

$$100p_f^a + 75p_c^a = 1000 + 1875 = 2875$$

c) The MRS is the same as before, but the MRT is different.

$$\frac{3F}{C} = \frac{p_C^a}{p_F^a} = \frac{25}{10}$$
$$C = \frac{6}{5}F$$

The new budget constraint is $25C + 10F = 2875$, so

$$25\frac{6}{5}F + 10F = 2875$$
$$F = \frac{2875}{40} = 71.875$$

and hence

$$C = \frac{2875 - 718.75}{25} = 86.25$$

d) Her utility in Cambridge is

$$U(75, 100) = \frac{1}{4}\ln 100 + \frac{3}{4}\ln 75 \approx 4.3894$$

and her utility in Palo Alto is

$$U(86.25, 71.875) = \frac{1}{4}\ln 71.875 + \frac{3}{4}\ln 86.25 \approx 4.4117$$

So her utility is greater in Palo Alto.

e) The budget lines for Cambridge and Palo Alto and relevant indifference curves are shown below: The budget line in Cambridge is

$$20C + 5F = 2000$$

and the budget line in Palo Alto is

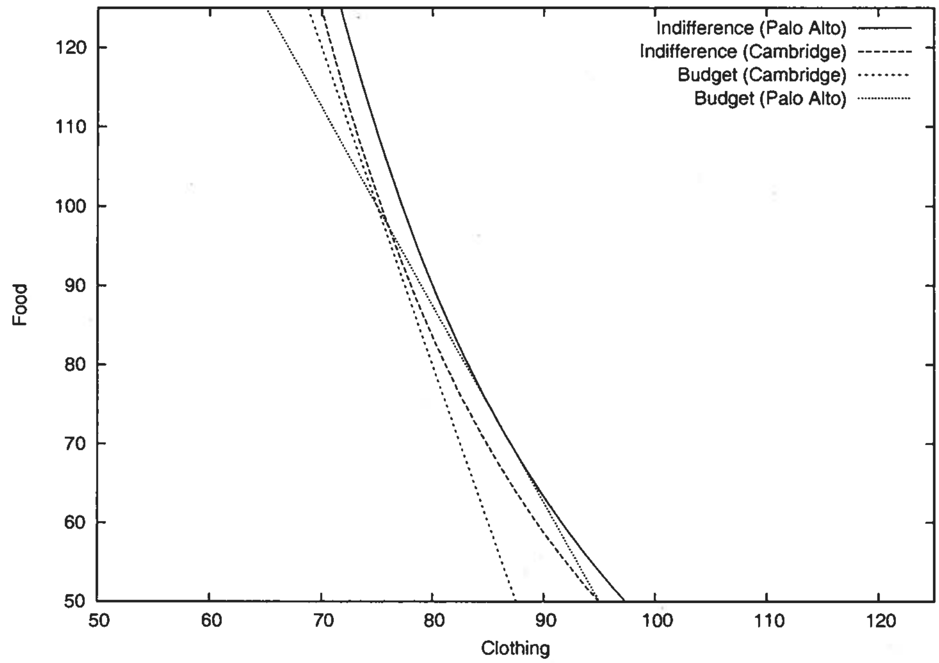
$$25C + 10F = 2875$$

The indifference curve in Cambridge is

$$\frac{1}{4}\ln F + \frac{3}{4}\ln C \approx 4.3894$$

and in Palo Alto

$$\frac{1}{4}\ln F + \frac{3}{4}\ln C \approx 4.4117$$



The budget line for Palo Alto intersects a higher indifference curve than the budget line for Cambridge, so utility is greater in Palo Alto.

Problem 4:

a) The marginal utility of coffee is

$$MU_C = \frac{\partial U}{\partial C} = \frac{D^{1/2}}{2C^{1/2}}$$

and the marginal utility of donuts is

$$MU_D = \frac{\partial U}{\partial D} = \frac{C^{1/2}}{2D^{1/2}}$$

so the marginal rate of substitution is

$$MRS = -\frac{MU_C}{MU_D} = -\frac{D}{C}$$

At the equilibrium point,

$$MRS = -\frac{D}{C} = -\frac{p_C}{p_D} = -\frac{\$1}{\$0.5}$$

$$D = 2C$$

So, applying the budget constraint $0.5D + C = I$,

$$(D, C) = (I, \frac{I}{2}) = (100, 50)$$

b) The income elasticity of demand for donuts is

$$\xi_D = \frac{\partial Q_D}{\partial I}\frac{I}{Q_D} = 1\frac{I}{I} = 1$$

and the income elasticity of demand for coffee is

$$\xi_C = \frac{\partial Q_C}{\partial I}\frac{I}{Q_D} = \frac{1}{2}\frac{I}{I/2} = 1$$

The income elasticities are both positive, so he purchases more of both goods as his income rises. The income elasticities are both 1, so the increase in his purchases of each good is proportional to the increase in his income.

c) If the price of coffee increases to \$2,

$$MRS = -\frac{D}{C} = -\frac{p_C}{p_D} = -\frac{\$2}{\$0.5}$$

$$D = 4C$$

and the budget constraint becomes $0.5D + 2C = I$,

$$(D, C) = (I, \frac{I}{4}) = (100, 25)$$

d) The bundle purchased was $(D, C) = (I, \frac{I}{2})$, which now equals $(200, 100)$.

e) The utility at the original equilibrium point is

$$U(50, 100) = 50^{1/2}100^{1/2} = 50\sqrt{2}$$

So the original indifference curve is $U(C, D) = 50\sqrt{2}$. The substitution effect point e^* is the point at which this indifference is tangent to the budget line parallel to the one after the shift. So we are tasked with finding the point at which $U(C, D) = 50\sqrt{2}$ and the imaginary budget line is parallel to the indifference curve.

$$MRS = -\frac{D}{C} = -\frac{p'_C}{p'_D} = -\frac{\$2}{\$0.5}$$

$$D = 4C$$

So

$$U(C, D) = C^{1/2}D^{1/2} = C^{1/2}(4C)^{1/2} = 50\sqrt{2}$$

$$C = 25\sqrt{2}$$

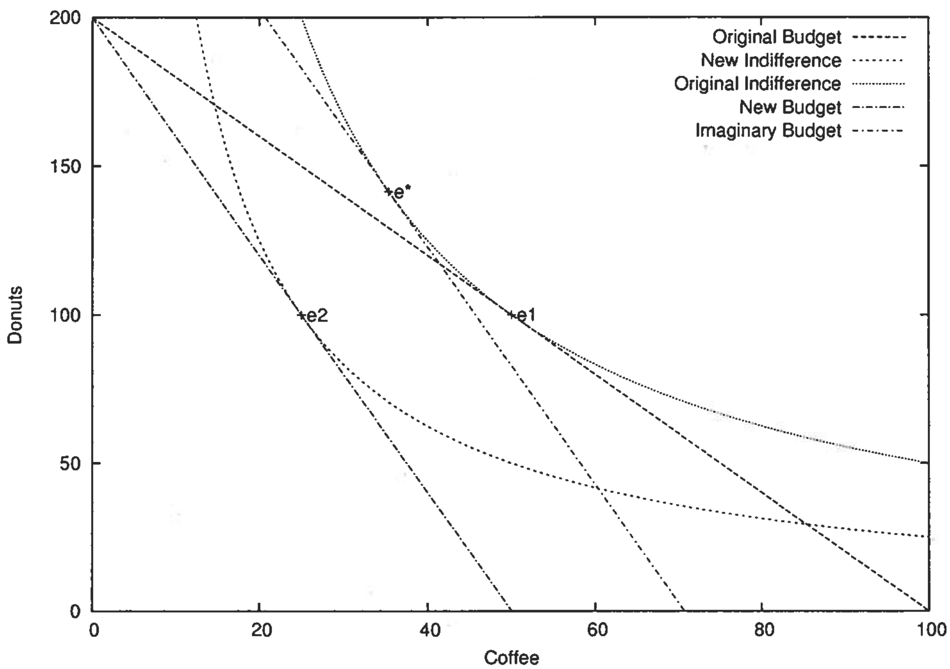
and

$$D = 100\sqrt{2}$$

So the substitution effect decreases the quantity of coffee consumed from 50 to $25\sqrt{2}$ (a decrease by a factor of $\sqrt{2}$), and decreases the quantity of donuts consumed from 100 to $50\sqrt{(2)}$, again a decrease by a factor of $\sqrt{2}$.

The income effect corresponds to a shift perpendicular to the budget line, from $(C, D) = (25\sqrt{2}, 50\sqrt{2})$ to $(25, 100)$.

f) The income and substitution effects are shown graphically below. The substitution effect moves the demand from point e_1 to e_*, and the income effect moves the demand from point e_* to e_2.



Problem 5:

Let a and b be goods for which an agent's preferences are given by $U(a, b) = 3a + 2b$. We show that neither good can be a Giffen good. To do so, we show that a is not a Giffen good; the argument for b is analogous. Note from the utility function that a and b are perfect substitutes. Thus, the quantity purchased of a is zero if $\frac{p_a}{p_b} > \frac{3}{2}$, and the full income is spent on $\frac{y}{p_a}$ units of a if $\frac{p_a}{p_b} < \frac{3}{2}$. Suppose a were a Giffen good. Then at some point, as the price p_a increased with all variables held constant, the quantity a purchased of a would increase. But this cannot happen. Consider first the case when the initial ratio of prices $\frac{p_a}{p_b} > \frac{3}{2}$. Then a is zero. If p_a increases, the consumer will still not purchase any of good a as their utility is maximized by purchasing only b. Alternatively, the initial ratio of prices could satisfy $\frac{p_a}{p_b} < \frac{3}{2}$. In this case, $a = \frac{y}{p_a}$. If p_a increases, it will be impossible to purchase this many or more units of a; it would violate the budget constraint. So a is a non-Giffen good. The same argument can be made for b. So neither good is a Giffen good.

Problem 6:

The initial bundle consumed is

$$(a, b) = (6, 14)$$

When the price of p_b is reduced to 5, and the consumer's income is reduced to maintain the same indifference curve, the bundle consumed is

$$(a, b) = (3, 18)$$

So the change due to substitution effects is $(3 - 6, 18 - 14) = (-3, 4)$.

And since the bundle consumed after the price change with the original income level is

$$(a, b) = (5, 30)$$

the change due income effects is $(5 - 3, 30 - 18) = (2, 12)$, or a scaling by a factor of $5/3$.