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Problem Set 6

Problem 1:

a) The inverse demand function is

$$p = 10 - Q_D$$

so the marginal revenue is

$$MR = p(Q) + Q\frac{\partial p}{\partial Q} = (10 - Q) + Q(-1) = 10 - 2Q$$

The firm maximizes profits at the quantity where marginal profit equals marginal costs, i.e.

$$MR = 10 - 2Q = 2 + Q = MC$$

$$Q = \frac{8}{3}$$

At this quantity, the price is $10 - \frac{8}{3} = \frac{22}{3} \approx 7.333$.

b) In a competitive market, the marginal cost is equal to the inverse demand curve.

$$10 - Q = 2 + Q$$

$$Q = 4$$

and so $p = 10 - 4 = 6$.

In a competitive market, the consumer surplus is

$$\int_0^{Q_e} p(Q_d) - p_e \, dQ_d = \int_0^4 10 - Q - 6 \, dQ = 8$$

and the producer surplus is

$$\int_0^{Q_e} p_e - MC(Q) \, dQ = \int_0^4 6 - (2 + Q) \, dQ = 8$$

so the welfare is 16.

In the monopoly situation, the consumer surplus is

$$\int_0^{Q_e} p(Q_d) - p_e \, dQ_d = \int_0^{\frac{8}{3}} 10 - Q - \frac{22}{3} \, dQ = \frac{32}{9}$$

and the producer surplus is

$$\int_0^{Q_e} p_e - MC(Q) \, dQ = \int_0^{\frac{8}{3}} \frac{22}{3} - (2 + Q) \, dQ = \frac{32}{3}$$

so the welfare is $\frac{128}{9}$.

So the deadweight loss is $16 - \frac{128}{9} = \frac{16}{9} \approx 1.777$.

c) To eliminate the deadweight loss, the government must provide a sufficient subsidy that the monopolist will be willing to produce the same quantity as would be produced in a competitive market equilibrium. This is $Q = 4$, $p = 6$.

The monopolist's marginal revenue is now $MR' = MR + \mathscr{S}$, where $\mathscr{S}$ is the amount of the subsidy. At the profit-maximizing point, $MR = MC$, so

$$10 - 2Q + \mathscr{S} = 2 + Q$$

$$3Q = 8 + \mathscr{S}$$

$$Q = \frac{8 + \mathscr{S}}{3}$$

We wish this quantity to be equal to the competitive market quantity, so

$$\frac{8 + \mathscr{S}}{3} = 4$$

$$\mathscr{S} = 4$$

The government must give a subsidy of 4 per unit.

Problem 2:

a) True. At the profit-maximizing choice of quantity and price, the producer surplus (the monopolist's profit) is maximized by definition. Total welfare is not maximized, since the consumer surplus is lower than it could be and there is a deadweight loss. However, increasing the consumer surplus would require decreasing the producer surplus, so this choice of price and quantity is Pareto-efficient.

b) True. Using the standard definition of social welfare as the sum of the consumer and producer surpluses, social welfare is the same. In a perfectly competitive market, the producer surplus is the area between the price line and the supply curve, and the consumer surplus is the area between the demand curve and the price line (both to the left of the equilibrium point). With a perfectly price-discriminating monopolist, the producer surplus is the area between the supply and demand curves (left of the equilibrium point). This is the same area. But even though the total welfare is the same, it has shifted all the surplus from the consumers to the producer, which is arguably detrimental to welfare.

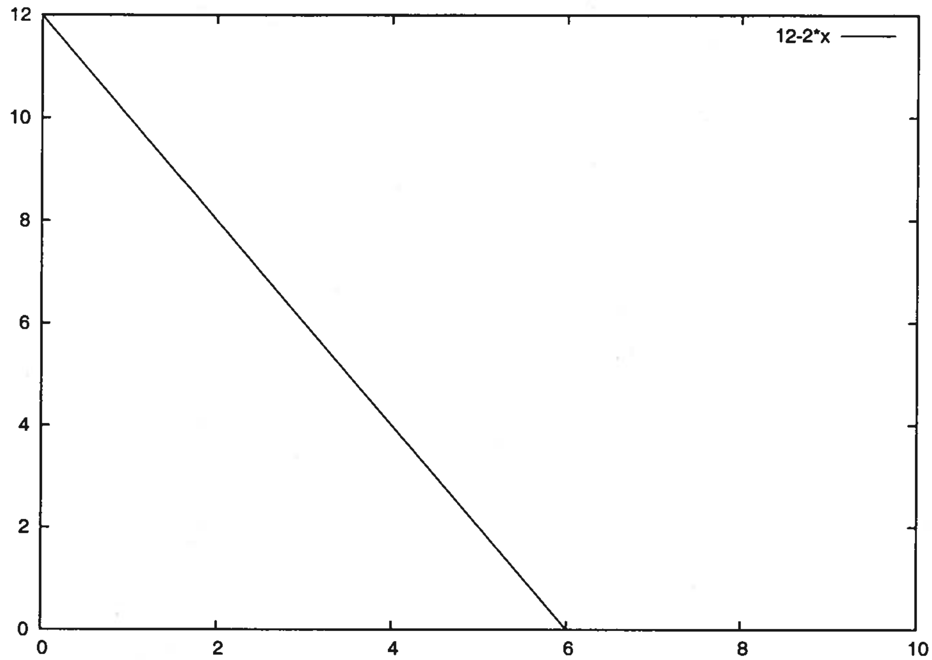
c) True. The marginal revenue can be written as

$$MR = \left(1 + \frac{1}{\epsilon}\right)$$

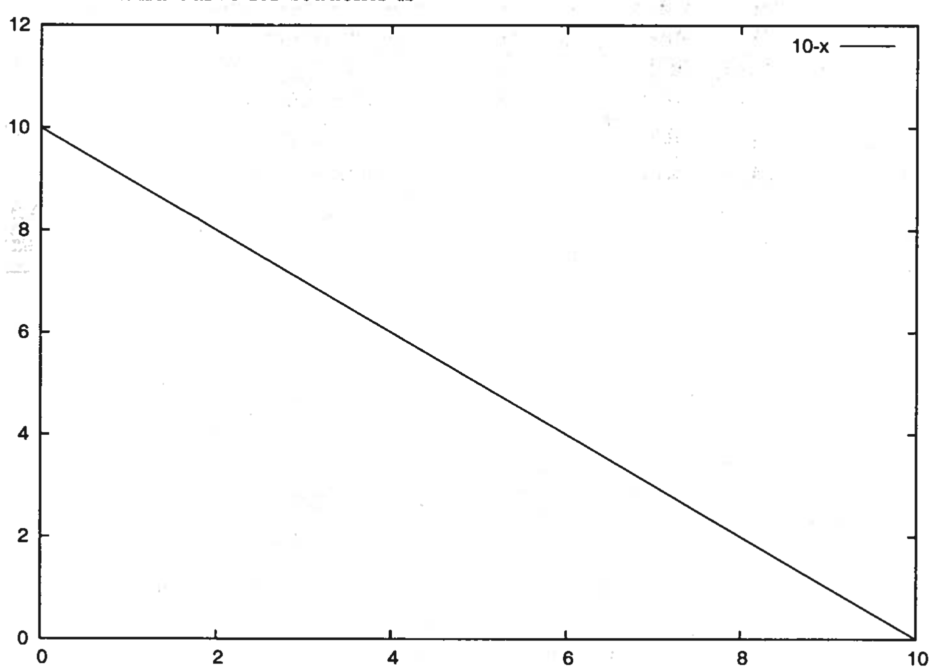
For maximum profits, the marginal revenue equals the marginal cost, which is zero in this case. So $\frac{1}{\epsilon} = -1$, or $\epsilon = -1$.

Problem 3:

a) The demand curve for workers is



and the demand curve for students is



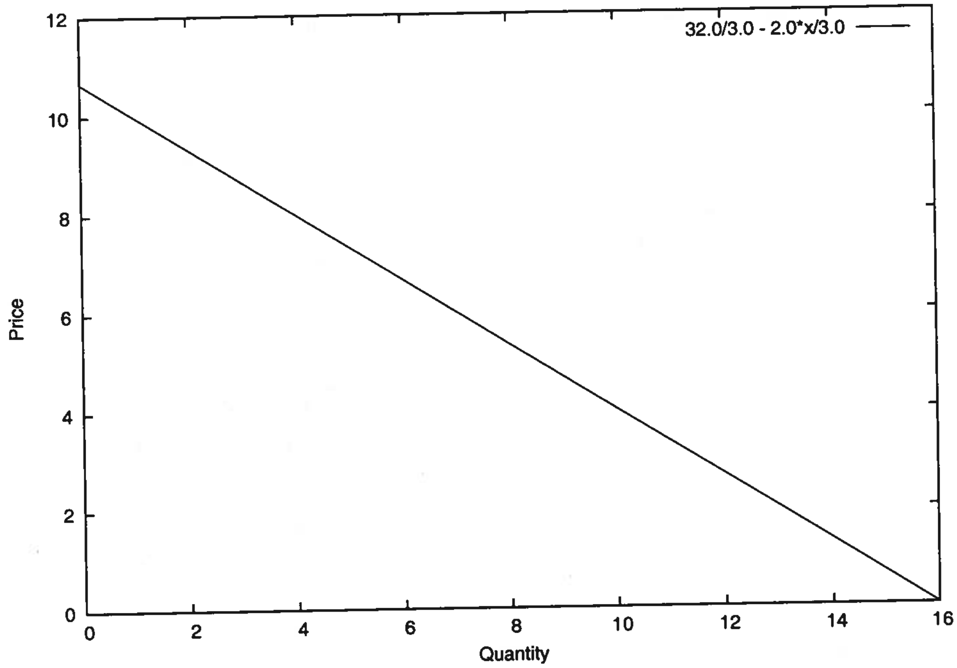
The total quantity demanded is the sum of that demanded by workers and students. The workers demand $Q_W = 6 - 1/2p$, and the students demand $Q_S = 10 - p$. So the total demand is

$$Q = Q_w + Q_s = 6 - 1/2p + 10 - p = 16 - 3/2p$$

The demand curve is thus

$$p = \frac{2}{3}(16 - Q)$$

This is shown below.



b) If the monopolist cannot distinguish the two types of consumers, it must charge them the same price, and demand is given by $Q_d = 16 - 3/2p$. The marginal revenue is

$$MR = p(Q) + Q\frac{\partial p}{\partial Q} = \frac{2}{3}(16 - Q) - \frac{2}{3}Q = \frac{32}{3} - \frac{4Q}{3}$$

The marginal cost is fixed at 5, so the profit maximizing choice has

$$\frac{32}{3} - \frac{4Q}{3} = 5$$

$$Q = \frac{17}{4} = 4.25$$

$$p = \frac{2}{3}(16 - Q) = \frac{2}{3}\left(16 - \frac{17}{4}\right) = \frac{47}{6} \approx 7.833$$

This gives a profit

$$\pi = Qp - 5Q = \frac{17}{4}\frac{47}{6} - 5\frac{17}{4} = \frac{799}{24} - \frac{85}{4} = \frac{289}{24} \approx 12.04$$

The quantity consumed by full-time workers is $6 - \frac{1}{2}\frac{47}{6} = \frac{25}{12}$. The quantity consumed by students is $10 - \frac{47}{6} = \frac{13}{6}$.

The consumer surplus for full-time workers is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{\frac{25}{12}} 12 - 2Q_D - \frac{47}{6} \, dQ_D = \frac{625}{144} \approx 4.34$$

The consumer surplus for students is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{\frac{13}{6}} 10 - Q_D - \frac{47}{6} \, dQ_D = \frac{169}{72} \approx 2.35$$

c) The full time workers have higher opportunity cost of time, so they will be less likely to spend the time to apply for a card. So the advantage of the cards is that the monopolist will be able to charge a lower price to students (with the card) than to the full-time workers.

d) The price can now be set independently for students and full time workers. For card holders (students), the marginal revenue is

$$MR = p(Q) + Q\frac{\partial p}{\partial Q} = 10 - Q + -1Q = 10 - 2Q$$

$$MR = 10 - 2Q = 5 = MC$$

The equilibrium quantity and price are

$$Q = 2.5$$

$$p = 10 - Q_D = 7.5$$

The consumer surplus for students is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{2.5} 10 - Q_D - 7.5 = 3.125$$

and the profit from sales to students is

$$\pi = pQ - 5Q = 6.25$$

For non-card holders (workers), the marginal revenue is

$$MR = p(Q) + Q\frac{\partial p}{\partial Q} = 12 - 2Q + -2Q = 12 - 4Q$$

$$MR = 12 - 4Q = 5 = MC$$

$$Q = 1.75$$

$$p = 12 - 2Q_D = 8.5$$

The consumer surplus for workers is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{1.75} 12 - 2Q_D - 8.5 = 3.0625$$

and the profit from sales to workers is

$$-2 \quad \pi = pQ - 5Q = 6.125$$

So the total profit is $6.25 + 6.125 = 12.\cancel{375}$.

e) Star Market makes a greater profit with the card (12.375 vs 12.04), so they are better off. The total consumer surplus without the cards is $4.34 + 2.35 = 6.69$, and with the cards it is $3.125 + 3.0625 = 6.1875$, which is less, so consumers are worse off.

Problem 4:

a) The inverse demand function is $p = 30 - Q$, and the marginal revenue for the monopolist is

$$MR = p(Q) + Q\frac{\partial p}{\partial Q} = 30 - Q + -1Q = 30 - 2Q$$

The marginal cost is

$$MC = \frac{\partial C}{\partial q} = q$$

so

$$30 - 2Q = Q$$
$$Q = 10$$
$$p = 30 - Q = 20$$

This is the profit-maximizing monopoly price. The profit to the monopolist is

$$\pi = pQ - C(q) = 200 - \frac{10^2}{2} = 150$$

b) The socially optimal price has $MC = p$, so

$$Q = 30 - Q$$
$$Q = 15$$
$$p = 30 - Q = 15$$

The profit to the monopolist (or producer surplus) is

$$\pi = pQ - C(q) = 225 - \frac{15^2}{2} = 112.5$$

The consumer surplus at the socially optimal price is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{15} 30 - Q_D - 15 = 112.5$$

So the social welfare is 225.

At the monopoly price the consumer surplus is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{10} 30 - Q_D - 20 = 50$$

So the social welfare in the monopoly situation is 200. The deadweight loss is 25.

c) Suppose a price ceiling is set at $p = 18$. The demand at this price is $30 - 18 = 12$. So the marginal revenue is as before for quantities above 12, and fixed at 18 for quantities below 12:

$$MR = \begin{cases} 30 - 2Q & Q > 12 \\ 18 & Q < 12 \end{cases}$$

The monopoly will maximize its profits by setting the price equal to the price ceiling. Then $p = 18$ and $Q = 12$. The profit of the monopolist is $pQ - C(Q) = 18(12) - \frac{12^2}{2} = 144$. The consumer surplus is

$$\int_0^{Q_e} p(Q_D) - p_e \, dQ_d = \int_0^{12} 30 - Q_D - 18 = 72$$

So the total welfare is 216, which gives a deadweight loss of 9.

d) To maximize total welfare, the government should set a price ceiling at the competitive price, 15. Then the monopolist will maximize profits by producing 15 units, at a profit of 112.5. This is the same outcome as in a competitive market, so the consumer and producer surpluses are both 112.5 (from part b), and there is no deadweight loss.

e) The business fee is a fixed cost, as it is independent of the quantity produced. So the marginal cost and marginal revenue are the same, and the profit maximizing outcome for the monopoly will be the same as in part a: $q = 10, p = 20$. Before the business fee, the profit is 150; the business fee reduces the profit to 20. But this is still a positive profit so the monopolist will not shut down.

With the price ceiling from part d, the profit is 112.5. This is positive, so the monopoly will choose to stay in business. [1]

f) With the price ceiling from part d, the profit is 112.5. This is less than 130, so when the business fee is subtracted, the monopolist will be losing money. So it will choose to shut down and avoid the business fee instead.

[1] I'm not entirely clear on what the problem was asking here. I'm assuming 4e is asking whether the monopoly will stay in business if the price ceiling is imposed with no business fee, and 4f is asking whether it'll stay in business with both the price ceiling and the business fee.