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14.01

Microeconomics

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Problem Set 7

Problem 1:

False. The firm that produces first will have the advantage because it knows what the second-producing firm's profit-maximizing output will be for any decision it makes. It can therefore choose its output to maximize its own profits, taking into account this information.

Problem 2:

False. For example, the perfectly competitive outcome can be forced, by government price regulation.

Problem 3:

a)

	advertise	don't
advertise	10,5	16,0
don't	6,8	10,2

If Firm A chooses to advertise, then Firm B will also choose to advertise, giving a payoff of 5. If Firm A chooses not to advertise, then Firm B will choose to advertise, giving a payoff of 8. So advertising is a dominant strategy for Firm B; whatever decision A makes, advertising will lead to more profit for B.

If Firm B chooses to advertise, then Firm A will choose to advertise, giving a payoff of 10. If Firm B chooses not to advertise, Firm A will choose to advertise, giving a payoff of 16. So advertising is also a dominant strategy for firm B.

b) Each player's choices holding the player's strategy constant are shown in bold below:

	advertise	don't
advertise	**10,5**	**16**,0
don't	6,**8**	10,2

The Nash equilibrium is where both players choose to advertise.

c) The payoff matrix now becomes:

	advertise	don't
advertise	**10,5**	16,0
don't	6,**8**	**20**,2

Firm B's dominant strategy is still to advertise. Firm A has no dominant strategy. The Nash equilibrium is still where both players choose to advertise.

Problem 4:

a) The market demand for widgets is $Q = 100 - \frac{p}{4}$. So the residual demand for firm A is $q_A = 100 - \frac{p}{4} - q_B$, and so $p = 400 - 4q_A - 4q_B$ and its marginal revenue curve is

$$MR = 400 - 8q_A - 4q_B$$

At the profit-maximizing point, the marginal revenue equals the marginal cost, which is constant at 100. So

$$400 - 8q_A - 4q_B = 100$$
$$q_A = 37.5 - \frac{q_B}{2}$$

By symmetry,

$$q_B = 37.5 - \frac{q_A}{2}$$

And so

$$q_A = 37.5 - \frac{37.5 - \frac{q_A}{2}}{2} = 37.5 - 18.75 + \frac{q_A}{4}$$
$$\frac{3}{4}q_A = 18.75$$
$$q_A = 25 = q_B$$

The equilibrium price is

$$p = 400 - 4q_A - 4q_B = 200$$

The profits for each firm are

$$\pi_A = \pi_B = 200(25) - 100(25) = 2500$$

b) This is a Stackelberg model, with Firm A as the leader. Firm B's best response will be

$$q_B = 37.5 - \frac{q_A}{2}$$

as before. Firm A thus faces a residual demand function of

$$p = 400 - 4q_A - 4q_B = 400 - 4q_A - 4(37.5 - \frac{q_A}{2}) = 400 - 4q_A - 150 + 2q_A = 250 - 2q_A$$

A's marginal revenue is

$$MR = p'(q_A)q_A + p(q_A) = 250 - 4q_A$$

To maximize profit, this equals the marginal cost of 100,

$$250 - 4q_A = 100$$
$$q_A = 37.5$$

and so

$$q_B = 37.5 - \frac{37.5}{2} = 18.75$$

and

$$p = 400 - 4(37.5) - 4(18.75) = 400 - 150 - 75 = 175$$

Firm A's profits are

$$\pi_A = pq_A - 100q_A = 2812.5$$

and Firm B's profits are

$$\pi_A = pq_B - 100q_B = 1406.25$$

c) The firms have the same marginal cost, so if they combine they will still have $MC = 100$. The marginal revenue is as in a monopoly situation

$$MR = p'(q)q + p(q) = 400 - 8Q_J$$

$$400 - 8Q_J = 100$$

$$Q_J = \frac{300}{8} = 37.5$$

$$p = 400 - 4Q_J = 250$$

$$\pi = pQ_J - 100Q_J = 9375 - 3750 = 5625$$

d) The total quantity to be sold to maximize joint profits is

$$Q_J = 37.5$$

from above, at a price $p = 250$. Firm A will give Firm B a market share of β, so B will produce $\beta 37.5$. Its profits will therefore be

$$(p - 100)Q_B = (250 - 100)\beta 37.5 = \beta 5625$$

This must be greater than the profits it would make in Cournot equilibrium for this to be preferable for B, so

$$\beta 5625 > 2500$$

$$\beta > \frac{2500}{5625} \approx 0.4444$$

So A must offer B at least 44.4% market share.

e) The price is 250, and the total quantity produced is 37.5. A produces 20.83 units and B 16.67 units. The profit for A is 3125, and the profit for B is 2500; the total profit is 5625. Thus B does not gain anything from the deal (at this market share), and A gains $3125 - 2500 = 625$; 625 is the total the firms gain jointly.

The consumer surplus without collusion is $\int_0^5 0400 - 4Q - 200\,dQ = 5000$. With the collusion, it is $\int_0^3 7.5400 - 4Q - 250\,dQ = 2812.5$, so consumers lose 2187.5 from the collusion.

f) Firm B will produce exactly $\frac{37.5}{2} = 18.75$ units. Thus the residual demand function for A is

$$p_A = 400 - 4q_A - 4q_B = 400 - 4q_A - 75 = 325 - 4q_A$$

A's marginal revenue will be

$$MR = 325 - 8q_A$$

Setting this equal to marginal cost gives

$$325 - 8q_A = 100$$

$$q_A = 28.125$$

$$p_A = 212.5$$

g) With Firm B in the market, Firm A makes a profit of 2500. Without B, A can make the full monopoly profit of 5625. So A will be willing to pay up $5625 - 2500 = 3125$ for B. Consumers would lose 2187.5 from the deal.

Problem 5:

a) Firm k faces a market demand curve of

$$p = a - b\sum_{i \neq k} q_i - bq_k$$

Its marginal revenue is

$$MR = a - b\sum_{i \neq k} q_i - 2bq)k$$

At the profit-maximizing point, this equals the marginal cost c

$$a - b\sum_{i \neq k} q_i - 2bq_k = c$$

b) Firm k's reaction function is given using the maximization shown above

$$a - b\sum_{i \neq k} q_i - 2bq_k = c$$

$$a - b\sum_{i \neq k} q_i - c = 2bq_k$$

$$q_k = \frac{a - b\sum_{i \neq k} q_i - c}{2b}$$

$$q_k = \frac{a-c}{2b} - \frac{\sum_{i \neq k} q_i}{2}$$

c) Since the firms have identical marginal costs, they will have the same quantity strategies for a Nash equilibrium. So

$$q_k = \frac{a-c}{2b} - \frac{\sum_{i \neq k} q_k}{2}$$

$$q_k = \frac{a-c}{2b} - \frac{n-1}{2} q_k$$

$$(n+1)q_k = \frac{a-c}{b}$$

$$q_k = \frac{a-c}{b(n+1)}$$

d)

$$p = a - bq = a - b\sum_{i} q_i = a - bnq_k$$

$$p = a - bn\frac{a-c}{b(n+1)}$$

$$p = a - \frac{n}{n+1}(a-c)$$

e) With free entry, the market price is the limit as n goes to infinity.

$$p = \lim_{n\to\infty} a - \frac{n}{n+1}(a-c) = c$$

The market price approaches the marginal cost, which is the price in a competitive equilibrium. This is the expected result, because when many firms can enter the market, the price-setting power of each individual firm is reduced.

Problem 6:

a) The firms produce identical products, so their Bertrand equilibrium is where price is equal to marginal cost: $p = MC = 1$. So the market demand is $Q = 2 - p = 1$

This is split between the two firms: $q_1 = q_2 = 1/2$. Neither firm makes a profit. The total welfare is the consumer surplus, which is $\int_0^1 2 - Q - 1\, dQ = 1/2$.

b) Suppose now that firm 1 commits to its price first. Let p be the price it chooses. Then there are three cases:

1. $p < 1$. This case will not happen: since the price is below the average variable cost, firm 1 will shut down instead of producing at this price.
2. $p = 1$. In this case, firm 2 will set its price to 1 as well (if they set a higher price, they have no sales; if they set a lower price, they take a loss.) So the two firms split the market with a price of 1 and a total quantity of 1, just as before.
3. $p > 1$. Then firm 2 will set its price just below p and take all of the sales. Firm 1 makes no profit.

Since the second case is the only one in which firm 1 makes a profit, it will always choose $p = 1$, and the output is the same as part a.

c) Suppose firm 2's cost falls to $0.5q_2$. Note that firm 1 still has cost of $1q_1$, so it will shut down for prices below 1. Thus, at any price below 1, firm 1 will shut down, and firm 2 will sell the entire market demand. So firm 2 sets its price just below 1 ($1 - \epsilon$ for some ϵ which we can make arbitrarily small, so in the limit we treat it as zero).

The quantity it sells is 1, as before, but now firm 2 makes the entire profit, and firm 1 produces nothing. Firm 1 now makes a profit of $1/2$, and the consumer surplus is still $1/2$, so total welfare is 1.

Problem 7:

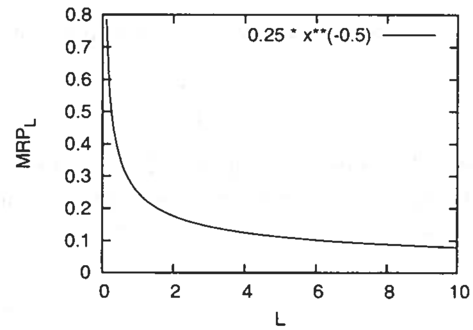
a) The marginal product of labor is

$$MP_L = \frac{\partial Q}{\partial L} = \frac{1}{4}L^{-1/2}$$

so the marginal revenue product of labor is

$$MPR_L = MR(MP_L) = pMP_l = \frac{1}{4}L^{-1/2}$$

This is graphed below:



b) Since the wage as a function of labor is $w(L) = L/2$,

$$ME = w(L) + L\frac{\partial w(L)}{\partial L} = \frac{L}{2} + L\frac{1}{2} = L$$

This equals the marginal revenue product of labor:

$$L = \frac{1}{4}L^{-1/2}$$

$$L^{3/2} = \frac{1}{4}$$

$$L = \left(\frac{1}{4}\right)^{2/3} \approx 0.397$$

and so

$$w = \frac{L}{2} \approx 0.198$$

c) The wage is now fixed at w, so the marginal expenditure of labor is $ME = w$. Now this equals the MRPL:

$$w = \frac{1}{4}L^{-1/2}$$

$$L_A = \frac{1}{16w^2}$$

This is the amount of labor used by firm A. The other three firms use the same amount of labor, so the total labor demand is four times this:

$$L = \frac{1}{4w^2}$$

The labor market equilibrium will have $L = 2w$, so

$$2w = \frac{1}{4w^2}$$

$$w = \frac{1}{2}$$

Wages increase because there is now competition among the four firms for labor.