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Microeconomics
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Problem Set 8

Problem 1:

a) False. The utilitarian SWF is the sum of each individual's utility, so this allocation may be optimal; the distribution does not matter.

b) False Suppose the two firms have linear PPFs with neither economies or diseconomies of scope. Then the firms are indifferent as to which combination of the two goods they will produce. So the total production in a Pareto-efficient allocation can be divided among the two firms differently.

c) False. After making the trade, there may still be further trades that will make both parties better off.

d) False. Avoiding a deadweight loss may lead to increased efficiency, but may sacrifice equity. Depending on the social welfare function used, a redistribution involving a deadweight loss may be preferable.

Problem 2:

a) The marginal product of labor is

$$MP_L = \frac{\partial Q}{\partial L} = \frac{1}{2\sqrt{L}}$$

In a perfectly competitive market, the marginal revenue is equal to the price, so $MR = 2$

$$MRP_L = MR(MP_L) = \frac{1}{\sqrt{L}}$$

b) Suppose the market wage for labor is 1/2. This equals the MRP_L, so

$$MRP_L = \frac{1}{\sqrt{L}} = \frac{1}{2}$$

$$L = 4$$

If the price of hot dogs increases to 3, the MRP_L becomes

$$MRP_L = 3\frac{1}{2\sqrt{L}}$$

Then

$$MRP_L = \frac{3}{2\sqrt{L}} = \frac{1}{2}$$

$$L = 9$$

c) The labor supply function is $L = 9w^2$, so

$$w = \sqrt{\frac{L}{9}} = \frac{\sqrt{L}}{3}$$

We now have

$$ME = w(L) + L\frac{\partial w(L)}{\partial L} = \frac{\sqrt{L}}{3} + L\frac{1}{2(3\sqrt{L})} = \frac{\sqrt{L}}{3} + \frac{\sqrt{L}}{6} = \frac{\sqrt{L}}{2}$$

The price of hot dogs is 2, so the $MRP_L = \frac{1}{\sqrt{L}}$ as before. So

$$MRP_L = \frac{1}{\sqrt{L}} = \frac{\sqrt{L}}{2} = ME$$
$$L = 2$$

d) In a competitive labor market, a minimum wage generally increases unemployment, but this is not the case in a monopsony labor market. The marginal expenditure curve becomes horizontal at the minimum wage over the region where the minimum wage is binding. If the minimum wage is set at the competitive level, it will intersect the MRP_L curve at the competitive equilibrium, thus increasing both the number of workers hired and their wage.

Problem 3:

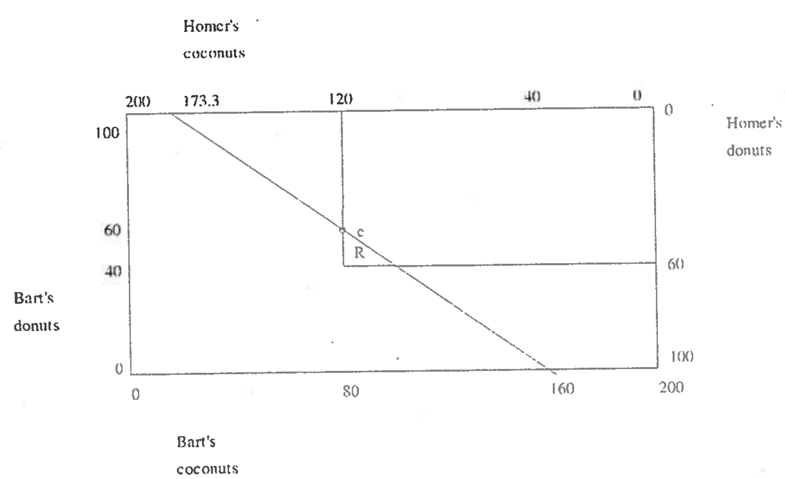
The initial endowment is shown as point e. Bart's utility is

$$U^B = \min\left(60, \frac{80}{2}\right) = 40$$

so his indifference curve is the Leontief curve with corner at $\langle 40, 80\rangle$, and Homer's utility is

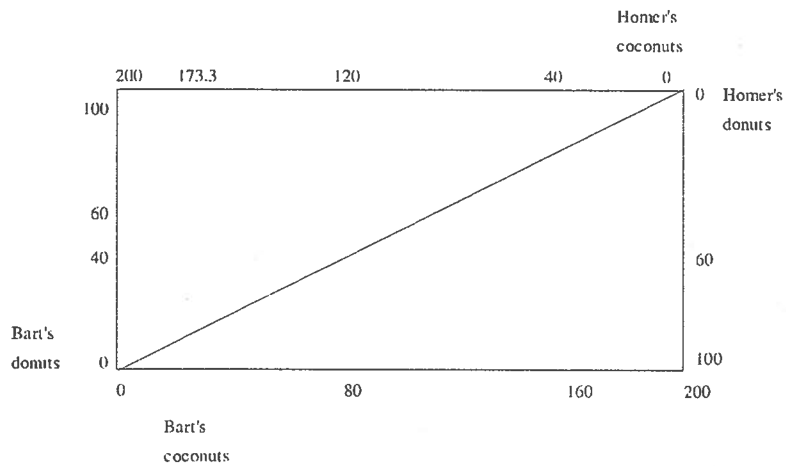
$$U^H = 4(40) + 3(120) = 160 + 360 = 520$$

so his indifference curve is the line between $\langle 100, 40\rangle$ and $\langle 0, \frac{520}{3}\rangle$. This is shown in the Edgeworth box below.



Both parties are better off with all bundles in region R.

The set of Pareto-efficient allocations are those in which the two parties are unwilling to trade. Bart has a Leontief utility function, so he will always be willing to trade except at the corner points. Thus, the contract curve is the set of all corner points:

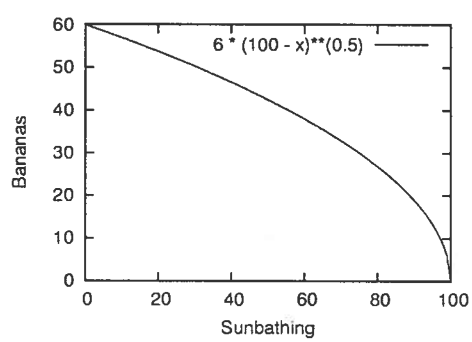


Problem 4:

a) Note that $H_B + H_S = 100$. Since $S = H_S$, $H_B = 100 - S$ and so

$$B = 6H_B^{1/2} = 6\left(100 - S\right)^{1/2}$$

b)



c) His utility is

$$U = 40B + BS = (40 + S)B = (40 + S)\left(6\sqrt{100 - S}\right)$$

At the maximum, marginal utility is zero:

$$\frac{dU}{dS} = 6\sqrt{100 - S} - \frac{3(40 + S)}{\sqrt{100 - S}} = 0$$

$$S = \frac{160}{3} \approx 53.333$$

$$B = 6\sqrt{100 - S} = 4\sqrt{105} \approx 40.988$$

d) The marginal rate of transformation is the slope of the PPF:

$$MRT = \left|\frac{dB}{dS}\right| = \frac{3}{\sqrt{100 - S}} = \frac{3}{\sqrt{100 - \frac{160}{3}}} = \frac{3\sqrt{105}}{70} \approx .439$$

Problem 5:

a) Alice's initial utility is

$$U_A = 7^{1/3}1^{2/5} \approx 1.912$$

So her indifference curve is

$$M_A^{1/3}C_A^{2/5} = \sqrt[3]{7}$$
$$M_A^{1/3} = \sqrt[3]{7}C_A^{-2/5}$$
$$M_A = 7C_A^{-6/5}$$

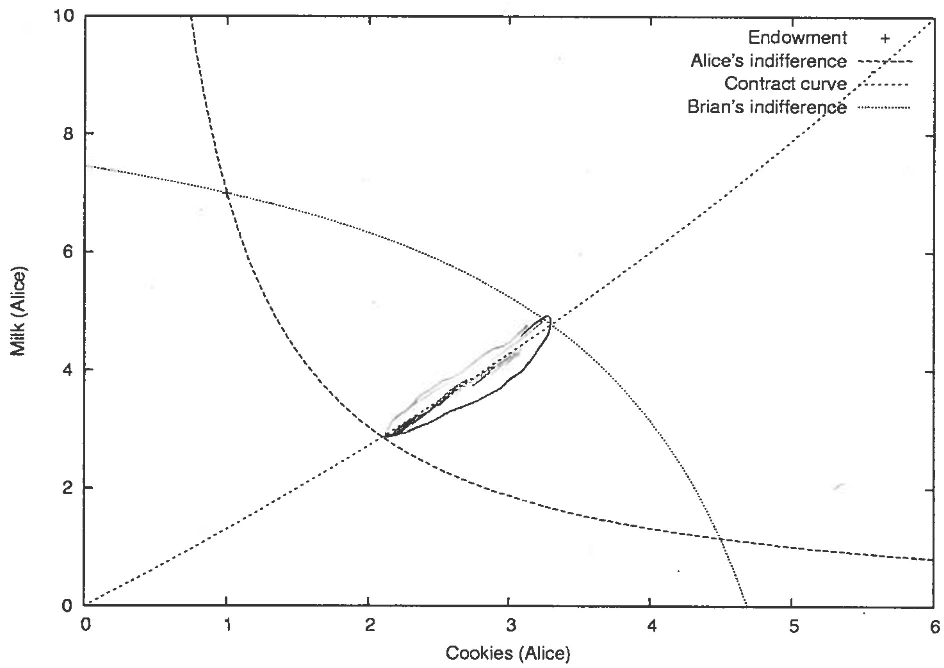
Brian's initial utility is

$$U_B = 3^{2/3}5^{3/5} \approx 5.4634$$

So his indifference curve is

$$M_B^{2/3}C_B^{3/5} = 3^{2/3}5^{3/5}$$
$$M_B^{2/3} = 3^{2/3}5^{3/5}C_B^{-3/5}$$
$$M_B = 3(5^{9/10})C_B^{-9/10} \approx 12.77C_B^{-9/10}$$

This is shown in the Edgeworth box below:



b) Alice's marginal utilities are

$$MU_{A,M} = \frac{\partial U_A}{\partial M_A} = \frac{1}{3} C_A^{2/5} M_A^{-2/3}$$

$$MU_{A,C} = \frac{\partial U_A}{\partial C_A} = \frac{2}{5} M_A^{1/3} C_A^{-3/5}$$

so her MRS is

$$MRS_A = \frac{MU_{A,M}}{MU_{A,C}} = \frac{5C_A}{6M_A}$$

Brian's marginal utilities are

$$MU_{B,M} = \frac{\partial U_B}{\partial M_B} = \frac{2}{3} C_B^{3/5} M_B^{-1/3}$$

$$MU_{B,C} = \frac{\partial U_B}{\partial C_B} = \frac{3}{5} M_B^{2/3} C_B^{-2/5}$$

so his MRS is

$$MRS_B = \frac{MU_{B,M}}{MU_{B,C}} = \frac{10C_B}{9M_B}$$

Recalling that Alice and Brian's bundles sum to 10 cups of milk and 6 cookies,

$$MRS_B = \frac{10C_B}{9M_B} = \frac{10(6 - C_A)}{9(10 - M_B)}$$

On the contract curve, Alice and Brian's MRSes are equal.

$$MRS_A = \frac{5C_A}{6M_A} = MRS_B = \frac{10(6 - C_A)}{9(10 - M_B)}$$

$$M_A = \frac{30C_A}{24 - C_A}$$

c) The allocations on the contract curve that represent mutual gains from trade are those points on the contract curve where utility is greater for both Alice and Brian. These are the points along the contract curve that are also in the region above Alice's indifference curve and below Brian's.

d) Now Alice's original indifference is

$$U_A = 6^{1/3}4^{2/5}$$

So her indifference curve is

$$M_A^{1/3}C_A^{2/5} = 6^{1/3}4^{2/5}$$
$$M_A^{1/3} = 6^{1/3}4^{2/5}C_A^{-2/5}$$
$$M_A = 64^{6/5}7C_A^{-6/5}$$

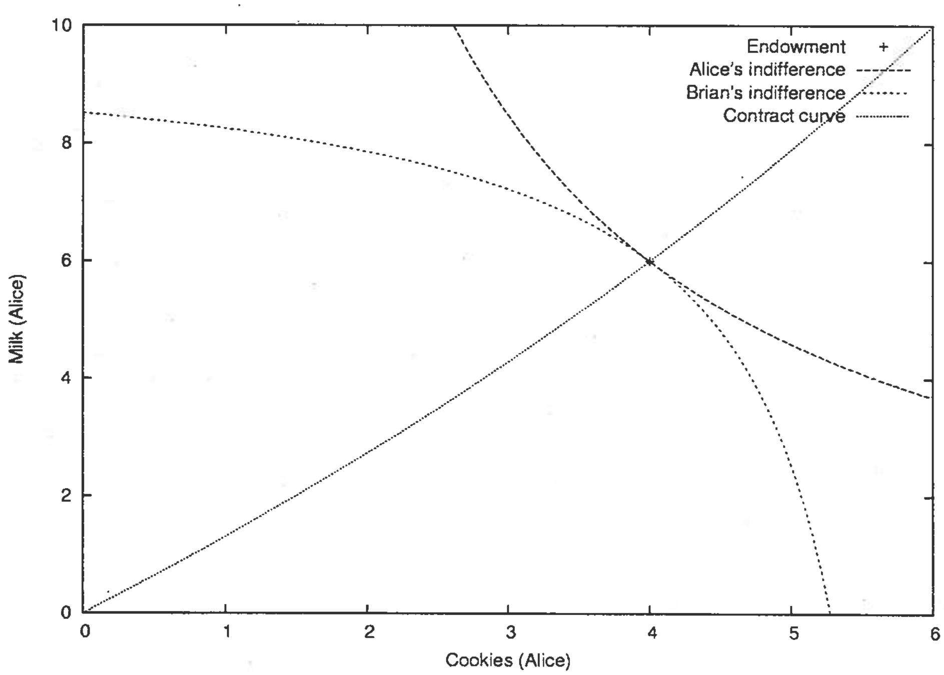
and Brian's new initial utility is

$$U_B = 4^{2/3}2^{3/5}$$

So his indifference curve is

$$M_B^{2/3}C_B^{3/5} = 4^{2/3}2^{3/5}$$
$$M_B^{2/3} = 4^{2/3}2^{3/5}C_B^{-3/5}$$
$$M_B = 4(2^{9/10})C_B^{-9/10}$$

The contract curve does not depend on the initial endowment, so it remains the same. The new Edgeworth box looks like



There are no trades that will make both parties better off.