(94)

14.01

Microeconomics

2004/12/03

Dan Ports

drkp@mit.edu

Problem Set 9

Problem 1:

a) True. For example, if either person has all 100 units, then there is no way to increase the other person's utility without decreasing the first person's.

b) False. This is true only in a competitive labor market. If the labor market is a monopsony, a minimum wage may increase both the wage and the employment level.

c) True. He is risk-neutral, so he will accept the gamble if his expected gain is positive. The expected value is

$$\frac{1}{4}(200) + \frac{3}{4}(-60) = 5$$

which is indeed positive.

d) False. It is possible that the cash flow will be extremely positive after the first three years, and that the utility due to this will outweigh the losses from the first three years. The exact amount required depends on the size of the losses in the first three years and the firm's utility function for wealth.

Problem 2:

a) No state is more Pareto-efficient than the others. In each of states A, B, and C, the other two states have less utility for at least one of the individuals.

b) Yes, D is Pareto-superior to A because the utilities for both are greater in D. It is not superior to B and C because one of the two utilities is higher in B and C (1 in B, 2 in C).

c) States B, C, D, and E are Pareto-optimal.

d) The social welfare is the sum of the individual's utilities:

State	U_1	U_2	W
A	50	50	100
B	70	40	110
C	45	54	99
D	53	50.5	103.5
E	30	84	114

So E is optimal.

e) This ratio is as small as possible when it equals 1, which is the case in state A, where the utilities are equal.

f) The optimal state with a Rawlsian welfare function is D, since the worse-off person's utility is highest.

g) The government faces an equity-efficiency tradeoff. State E is more efficient in that the total utility is highest, but is also the least equitable in that individual's 1 utility is so much lower than individual 2's.

h) Note that E is the more efficient outcome, but A is more equitable. The welfare in case E is

$$W_A = \alpha(50) + (1-\alpha)(50) = 50$$

In outcome E, the welfare is

$$W_E = \alpha(30) + (1-\alpha)(84) = 84 - 54\alpha$$

The less efficient outcome, outcome A, is chosen as long as $W_A > W_E$, i.e.

$$\begin{aligned} 50 &> 84 - 54\alpha \\ \alpha &> \frac{17}{27} \approx 0.63 \end{aligned}$$

So if $0.5 \leq \alpha < 0.63$, the more efficient outcome is chosen.

i) Individual 1 prefers state B, and individual 2 prefers state A. If individual 1 transfers 10 units to individual 2, then 2 will be indifferent between B and A.

Problem 3:

Sarah's MRS is

$$MRS_S = \frac{MU_{X,S}}{MU_{Y,S}} = \frac{\frac{\partial U_S}{\partial X_S}}{\frac{\partial U_S}{\partial Y_S}} = \frac{20Y_S^{0.6}X_S^{-0.6}}{30X_S^{0.4}Y_S^{-0.4}} = \frac{2}{3}\frac{Y_S}{X_S}$$

and Jane's MRS is

$$MRS_J = \frac{MU_{X,J}}{MU_{Y,J}} = \frac{\frac{\partial U_J}{\partial X_S}}{\frac{\partial U_J}{\partial Y_S}} = \frac{50Y_J^{0.5}X_J^{-0.5}}{50X_J^{0.5}Y_J^{-0.5}} = \frac{Y_J}{X_J}$$

We also have the conditions

$$\begin{aligned} X_S + X_J &= 58 \\ Y_S + Y_J &= 36 \end{aligned}$$

At equilibrium, the MRSes are equal. So

$$\begin{aligned} MRS_S &= MRS_J \\ \frac{2Y_S}{3X_S} &= \frac{Y_J}{X_J} \\ \frac{2Y_S}{3X_S} &= \frac{36 - Y_S}{58 - X_S} \\ Y_S &= \frac{108X_S}{X_S + 116} \end{aligned}$$

b)?. −6

Problem 4:

a) The initial allocation is $\langle A,\ B,\ C \rangle = \langle 400,\ 100,\ 16 \rangle$, so the initial social welfare is

$$W = \sqrt{F_A} + \sqrt{F_B} + \sqrt{F_C} = \sqrt{400} + \sqrt{100} + \sqrt{16} = 20 + 10 + 4 = 34$$

The initial individual utilities are

$$\langle U_A,\ U_B,\ U_C \rangle = \langle 20,\ 10,\ 4 \rangle$$

After redistribution, the allocation is $\langle A,\ B,\ C\rangle = \langle 225,\ 196,\ 16\rangle$. So the social welfare is

$$W = \sqrt{F_A} + \sqrt{F_B} + \sqrt{F_C} = \sqrt{225} + \sqrt{196} + \sqrt{16} = 15 + 14 + 4 = 33$$

The individual utilities are

$$\langle U_A,\ U_B,\ U_C\rangle = \langle 15,\ 14,\ 4\rangle$$

Social welfare decreases slightly. Intuitively, this is because the loss of food due to spoilage outweighs the greater utility that B gains from the food compared to A.

b) Now the allocation is $\langle A,\ B,\ C\rangle = \langle 225,\ 100,\ 100\rangle$. The social welfare is

$$W = \sqrt{F_A} + \sqrt{F_B} + \sqrt{F_C} = \sqrt{225} + \sqrt{100} + \sqrt{100} = 15 + 10 + 10 = 35$$

The individual utilities are

$$\langle U_A,\ U_B,\ U_C\rangle = \langle 15,\ 10,\ 10\rangle$$

c) The social welfare is higher in (b) than in the initial allocation and in the redistribution in (a), despite the fact that there is a greater loss due to spoilage in (c). This is because C initially has much less food than A, C's marginal utility is sufficiently higher than A's that C's increase in utility will outweigh both A's decrease in utility and the loss due to spoilage.

d) For a fixed total amount of food, in this case utility is maximized when all three individuals divide the food equally. This is not true in general of utilitarian social welfare functions; it is only true here because each individual has the same utility function with diminishing returns.

Problem 5:

a) If the backpacker keeps all of her money, she has a $1/2$ chance of having zero dollars and a $1/2$ chance of having \$600. So her expected wealth is \$300.

If she puts half of her money in the bush, she has a $1/4$ chance of having zero dollars, a $1/2$ chance of having \$300, and a $1/4$ chance of having \$600. The expected wealth again is \$300.

If the backpacker is risk neutral, her marginal utility of wealth is constant and she will choose based on expected value. So she is indifferent to the two options.

b) Now if she keeps all of her money, her expected utility is

$$\frac{1}{2}\sqrt{600} \approx 12.25$$

If she puts half in the bush, her expected utility is

$$\frac{1}{2}\sqrt{300} + \frac{1}{4}\sqrt{600} \approx 14.78$$

So she will choose to put half of her money in the bush.

c) Her expected utility if she keeps all her money is

$$\frac{1}{2}600^\alpha = 300^\alpha\left(\frac{2^\alpha}{2}\right)$$

If she puts half in the bush, it is

$$\frac{1}{2}300^\alpha + \frac{1}{4}600^\alpha = 300^\alpha\left(\frac{1}{2} + \frac{2^\alpha}{4}\right)$$

She will only choose to keep all her money if

$$300^\alpha\left(\frac{2^\alpha}{2}\right) > 300^\alpha\left(\frac{1}{2}+\frac{2^\alpha}{4}\right)$$

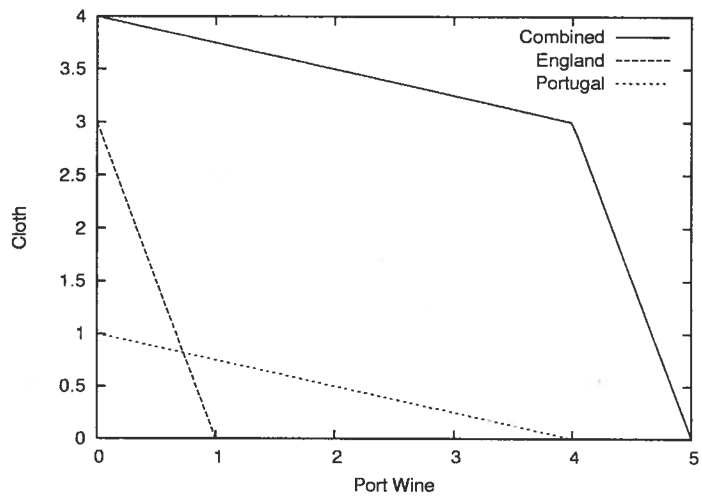
$$2^\alpha > 2$$

which is only true if $\alpha > 1$.

(There is also the corner case where $\alpha = 0$. In this case, she has no utility of wealth; thus, she is utterly indifferent to how much money she has, and does not care which option she chooses.)

Problem 6:

a) The production possibility frontiers are shown below.



England has a comparative advantage in the cloth industry and Portugal has a comparative advantage in the port industry. The relative price of cloth in terms of wine is 1/3 in England and 4 in Portugal.

b) England has the comparative advantage in the cloth industry, so it is better off when the relative price of cloth in terms of port is less than 4. Portugal is better off when the relative price is greater than 1/3. So only Portugal gains for prices between 0 and 1/3, both gain for prices between 1/3 and 4, and only England gains for prices above 4.