

**Massachusetts Institute of Technology  
Department of Economics**

**14.01 Principles of Microeconomics  
Midterm #1  
Tuesday, October 5, 2004**

Please print legibly.

Last Name: PORTS First Name: DAW  
Student ID: 968 819 927

**IMPORTANT:** Please circle your section.

Edward Cho	10 AM	1 PM	2 PM
Saugato Datta	11 AM	1 PM	
Alan Grant	10 AM	11 AM	<u>12 PM</u>
Ummul Hasanath Ruthbah	9 AM		2 PM

**Instructions:**

You have two hours to complete this exam. The test has a total of 100 points on 10 pages. You should try to complete each section in 30 minutes leaving 30 minutes at the end to check your answers. You may use a non-programmable calculator. We encourage you to write your answers in pen as it facilitates any regrading requests you may have.

**Do not write anything in the area below.**

Question 1: 5 /5    Question 7(a): 7 /7    Question 8(a): 12 /12  
 Question 2: 0 /5    Question 7(b): 7 /7    Question 8(b): 11 /11  
 Question 3: 4 /5    Question 7(c): 7 /7    Question 8(c): 10 /12  
 Question 4: 5 /5    Question 7(d): 7 /7  
 Question 5: 5 /5    Question 7(e): 7 /7  
 Question 6: 5 /5  
 Question 7: 35 /35  
 Question 8: 33 /35  
 TOTAL: 92 /100



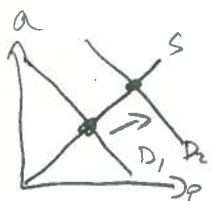
### Section I: True/False/Uncertain

In this section, write whether each statement is true, false, or uncertain. Please fully explain your answer, using a diagram if appropriate. No credit will be given for an answer without an explanation.

- (5 points) In a particular market, equilibrium price and quantity both increase. So the law of demand does not work here.

False

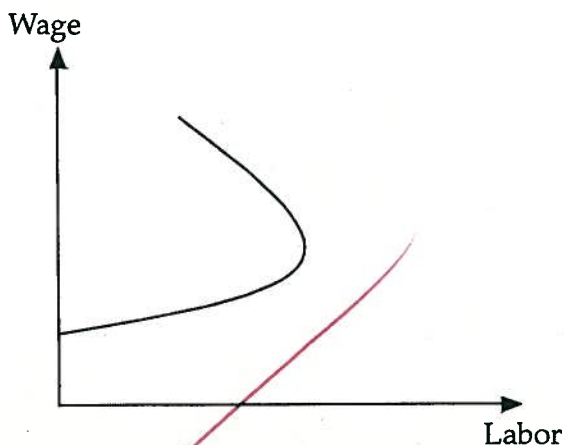
The law of demand states that demand decreases as price increases, i.e. that demand curves are downward-sloping. Assuming a downward-sloping demand curve, a rightward shift of the demand curve due to some externality can increase both equilibrium price and quantity. So this does not require the law of demand to be



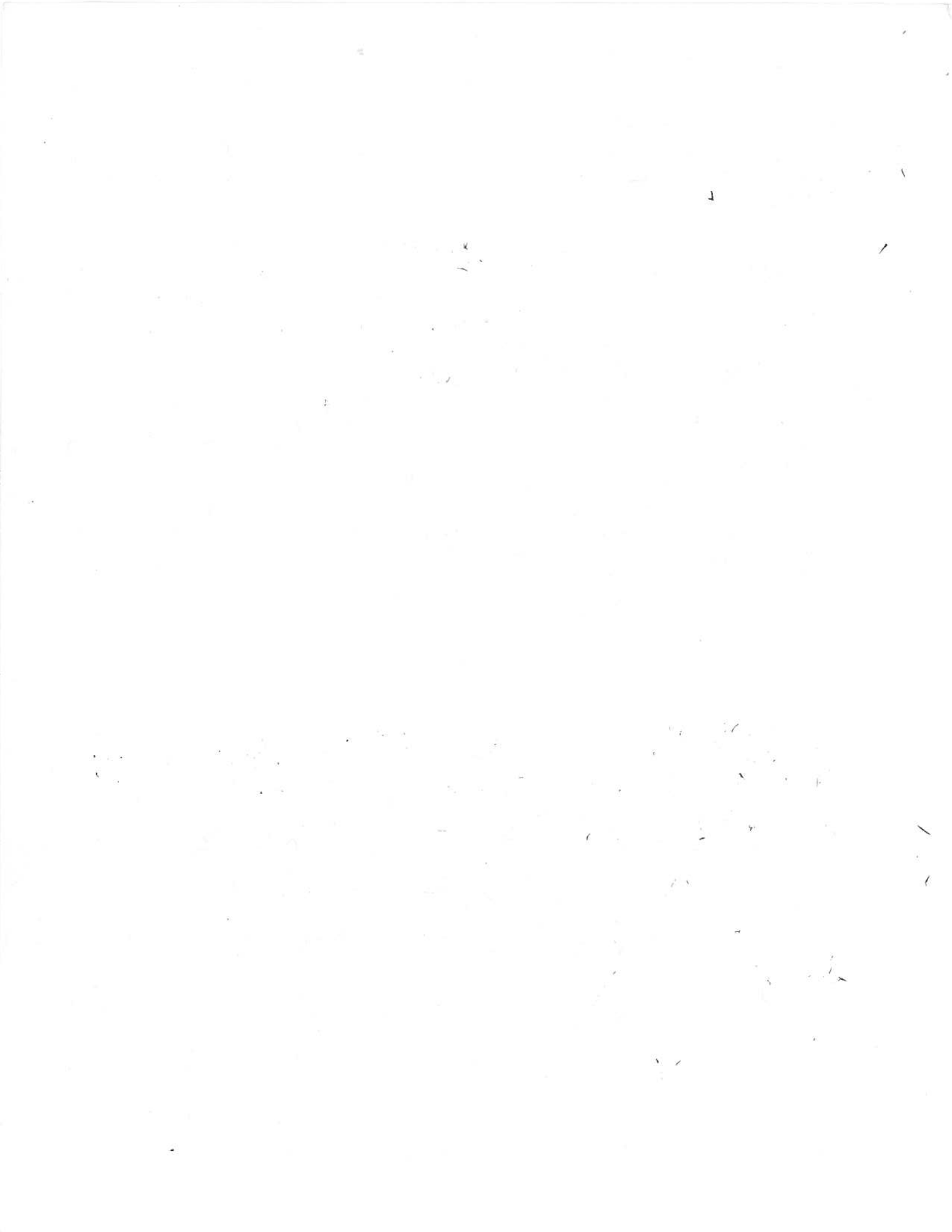
- (5 points) Bob has the following labor supply curve. It implies that leisure is a normal good ~~violated~~ only at the upward sloping part of the supply curve.

False

0



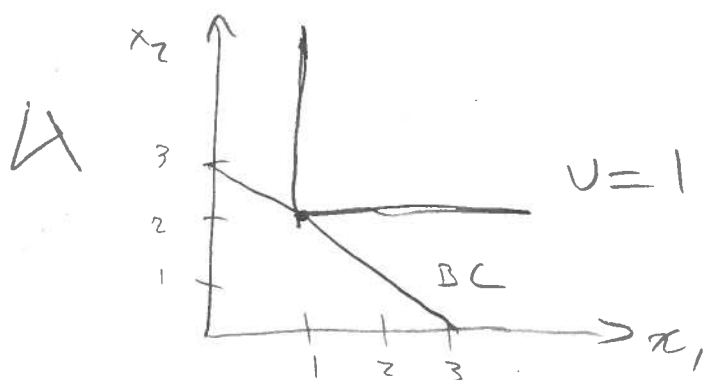
In the upward sloping part of the curve, as wage increases labor increases  $\Rightarrow$  leisure decreases so leisure is an inferior good in this region.



3. (5 points) Assume that Amy has a leontief utility function (perfect complements) and the optimal bundle for  $(x_1, x_2)$  is  $(1, 2)$ . We know that the price ratio is  $\frac{1}{2}$  since she consumes the goods in these proportions.

False

Consider the counterexample  $U = \min \{x_1, \frac{1}{2}x_2\}$ .  
 Suppose the prices are  $p_1 = p_2 = 1$  and her budget is 3. The optimal bundle will be  $(1, 2)$ , but the price ratio is not  $\frac{1}{2}$ .



*the example is not correct.*

4. (5 points) Loews movie theatre estimates the elasticity of demand with respect to price for movies to be  $-\frac{1}{2}$ . As a result, Loews concludes that it can increase its price indefinitely since demand is inelastic and make infinite revenues.

The elasticity of demand,  $\epsilon = \frac{\partial Q}{\partial P} \frac{P}{Q}$ , is computed at a particular price, so it may change as the price is increased. When  $\epsilon = \frac{1}{2}$ ,  $R'(P) = q(1 - |\epsilon|) > 0$ , so increasing the price increases revenues. If the demand elasticity is  $-\frac{1}{2}$  at all prices then they will be able to increase their prices indefinitely, otherwise there is a limit.

False



5. (5 points) If John considers Pepsi and Coke as perfect substitutes, then he chooses any bundle on his budget constraint since one good is just as good as the other.

False.

This is only true if they have the same prices. If Coke costs twice as much as Pepsi, then he will be able to purchase  $x$  units of Coke or  $2x$  units of Pepsi. Both of these are on the budget constraint, but twice as much Pepsi is preferable.

- 5 6. (5 points) The CPI went up from 180 in 2003 to 270 in 2004. Consumers are worse off by 50%.

False.

This does not take into account the substitution effect. Consumers will pay 50% more for the same bundle of goods they purchased in 2003, but because of the new prices they will select a different bundle that maximizes utility.



## Section II: Long Answer Question #1

This question is in parts and is cumulative. If you cannot solve part of the question and you need the results of that part in later parts, you should assume a generic functional form (e.g.  $q_S(p) = S(p)$ ) or describe what you would do with the solution from the earlier part to receive partial credit.

7. Consider the following market for cars,

$$Q^D = 20000 - 20P_T + 10P_H + 5Y$$

$$Q^S = 40000 + 40P_T$$

where  $Q^D$  and  $Q^S$  are the demand and supply for Toyota,  $P_T$  is the price for Toyota,  $P_H$  is the price of Hyundai and  $Y$  is the income of consumers.

(a) (7 points) Find the market equilibrium for Toyota cars when  $P_H = \$12000$  and  $Y = \$100000$ .

7

$$Q^S = Q^D$$

$$40000 + 40P_T = 20000 - 20P_T + 10P_H + 5Y$$

$$20000 + 60P_T = 120000 + 500000$$

$$60P_T = 600000$$

$$P_T = 10,000$$

$$Q^D = 20000 - 20P_T + 10P_H + 5Y$$

$$= 20000 - 200000 + 120000 + 500000$$

$$Q = 440,000$$



- (b) (7 points) What are the supply and demand elasticities when the market is in equilibrium? What is the income elasticity of demand? What is the cross-price elasticity between Toyota and Hyundai?

7

Demand elasticity =  $\frac{\partial Q^D(P_T)}{\partial P_T} \frac{P_T}{Q^D(P_T)} = -20 \frac{P_T}{Q^D(P_T)} = -20 \frac{10000}{440000} = -\frac{5}{11}$

Supply elasticity =  $\frac{\partial Q^S(P_T)}{\partial P_T} \frac{P_T}{Q^S(P_T)} = 40 \frac{P_T}{Q^S(P_T)} = 40 \frac{10000}{440000} = \frac{10}{11}$

Income elasticity =  $\frac{\partial Q^D(P_T)}{\partial Y} \frac{Y}{Q^D(P_T)} = 5 \frac{Y}{Q^D(P_T)} = 5 \frac{100000}{440000} = \frac{50}{44} = \frac{25}{22}$

Cross-price elasticity =  $\frac{\partial Q^D(P_T)}{\partial P_H} \frac{P_H}{Q^D(P_T)} = 10 \frac{P_H}{Q^D(P_T)} = 10 \frac{12000}{440000} = \frac{3}{11}$

- 7 (c) (7 points) Suppose the Toyota dealers now increase the price of Toyotas to \$11000. How will the revenue change? Explain why the revenue changes in this direction. How much revenue they will earn?

At the initial price,  $R = QP = 440,000 \cdot 10,000 = 4,400,000,000$ .

With  $P_T = 11000$ ,  $Q_D = 20000 - 20P_T + 10P_H + 5Y$

$Q_D = 20000 - 20 \cdot 11000 + 10 \cdot 12000 + 5 \cdot 100000$

$= 420,000$

So  $R' = Q'P' = 420000 \cdot 11000 = 4,620,000,000$ .

The revenue increases. This is because the demand elasticity has magnitude less than 1; so the relative decrease in demand is smaller than the relative increase in price.



- (d) (7 points) Suppose the government imposes a tax of \$1000 per Toyota sold. What is the new equilibrium in the market for Toyotas? Show mathematically and graphically. How much tax revenue does the government collect?

Suppose the tax is imposed on the seller, i.e.

$$Q_S'(P_T) = Q_S(P_T - 1000)$$

$$Q_S'(P_T) = Q_D(P_T) \Rightarrow Q_S(P_T - 1000) = Q_D(P_T)$$

$$40000 + 40(P_T - 1000) = 20000 - 20P_T + 10P_H + 54$$

$$40000 + 40P_T - 40000 = 20000 - 20P_T + 10 \cdot 12000 + 5 \cdot 100000$$

$$60P_T = 640000 \Rightarrow P_T \approx 10666.6$$

$$Q = Q_S = 40000 + 40(10666.6 - 1000) = 426666.6$$

The government collects 1000  $Q = \$426,666,666.6$

graphical solution over  $\rightarrow$

- (e) (7 points) If the price of Hyundai decreases to \$9000 how much revenue does the government collect? How does this relate to part (d)?

$$Q_S'(P_T) = Q_D(P_T)$$

$$40000 + 40P_T - 4000 = 20000 - 20P_T + 10P_H + 54$$

$$60P_T = 20000 + 90000 + 500000$$

$$60P_T = 610,000 \Rightarrow P_T \approx 10166.6$$

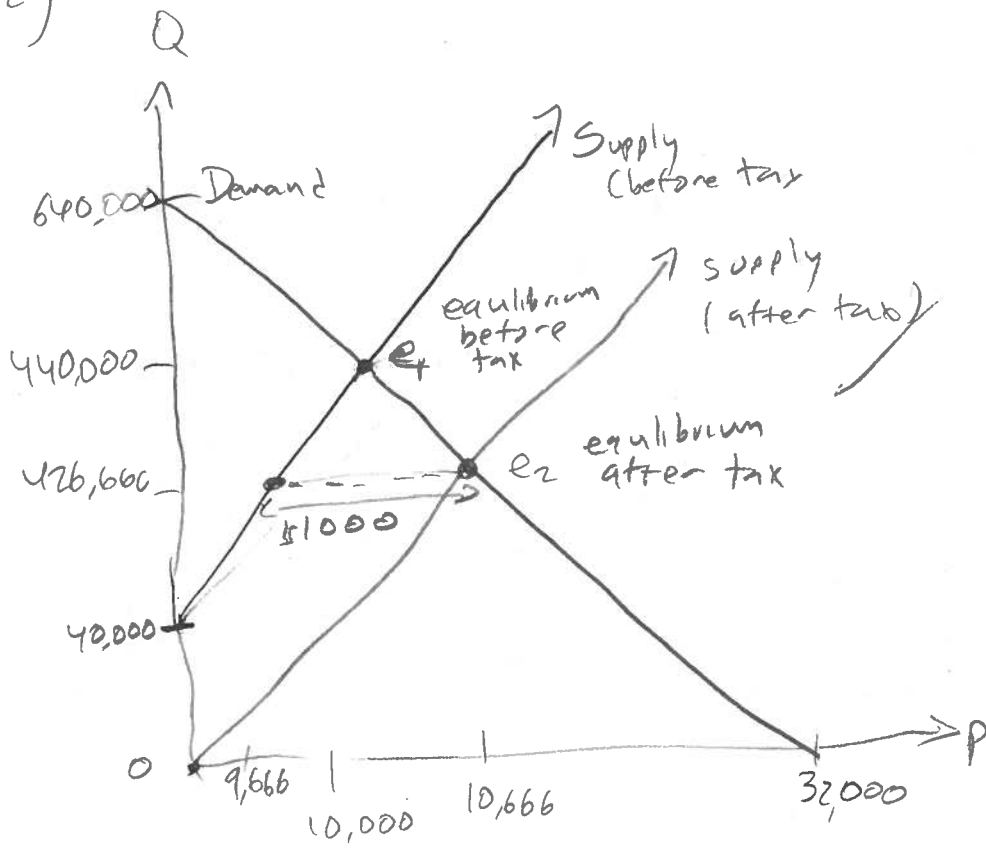
$$Q = Q_S = 40000 + 40(10166.6 - 1000) = 406666.6$$

The government collects 1000  $Q = 406666.6$

$$= \$406,666,666.6$$

The price of a substitute has fallen, so the demand curve shifts left, lowering the equilibrium price and quantity. So the government's revenue falls.

7d)



(not to scale)

### Section III: Long Answer Question #2

This question is in parts and is cumulative. If you cannot solve part of the question and you need the results of that part in later parts, you should assume a generic functional form (e.g.  $q_s(p) = S(p)$ ) or describe what you would do with the solution from the earlier part to receive partial credit.

8. Susan has the following utility function for leisure ( $L$ ) and consumption ( $C$ ):

$$U = L^{1/2}C^{1/2} + 10.$$

She works at a movie theater at an hourly wage of  $\$W$ . The price of consumption is  $\$P$  per unit.

- (a) (12 points) How many hours does she work and how much does she consume?

Budget constraint:  $PC + WL = 24W$

$$\frac{MU_L}{MU_C} = \frac{\partial U}{\partial L} = \frac{C^{1/2}}{2L^{1/2}} \quad MU_C = \frac{\partial U}{\partial C} = \frac{L^{1/2}}{2C^{1/2}}$$

$$MRS = -\frac{MU_L}{MU_C} = \frac{\frac{C^{1/2}}{2L^{1/2}}}{\frac{L^{1/2}}{2C^{1/2}}} = \frac{C}{L} = -\frac{W}{P}$$

$$\Rightarrow CP = WL$$

$$CP + CP = 24W \Rightarrow 2CP = 24W \Rightarrow C = \frac{12W}{P}$$

$$WL + WL = 24W \Rightarrow L = 12$$

So she works 12 hrs per day

and consumes  $\frac{12W}{P}$  units.

12



- (b) (11 points) Susan also consumes  $(x_1, x_2)$  where  $x_1$  is movies and  $x_2$  is steaks and her utility function over these goods is given by:

$$U(x_1, x_2) = e^{x_1^2 x_2}$$

Suppose prices are  $(p_1, p_2) = (6, 3)$  and the wage Susan receives from the movie theater is  $W = 5$ . Susan spends all of her income from the movie theater on movies and steaks. How many movies and steaks does she consume? (If you could not solve part (a) for her labor decision, let  $K =$  number of hours she works).

$$MU_1 = \frac{\partial U}{\partial x_1} = 2x_1 x_2 e^{x_1^2 x_2}$$

$$MU_2 = \frac{\partial U}{\partial x_2} = x_1^2 e^{x_1^2 x_2}$$

$$MRS = -\frac{MU_1}{MU_2} = -\frac{2x_1 x_2 e^{x_1^2 x_2}}{x_1^2 e^{x_1^2 x_2}} = -\frac{2x_2}{x_1}$$

At an interior soln,  $MRS = -\frac{p_1}{p_2}$

$$\Rightarrow -\frac{2x_2}{x_1} = -\frac{6}{3} \Rightarrow 6x_2 = 6x_1 \Rightarrow x_1 = x_2$$

The budget constraint is

$$p_1 x_1 + p_2 x_2 = wK$$

$$\Rightarrow 6x_1 + 3x_2 = 5K$$

$$9x_1 = 5K \Rightarrow x_1 = \frac{5}{9}K = \frac{5}{9}12 = 20/3$$

$$9x_2 = 5K \Rightarrow x_2 = \frac{5}{9}K = \frac{5}{9}12 = 20/3$$

She consumes  $20/3$  units each of movies and steak.



- (c) (12 points) In response to excess demand, the movie theater decides to increase the price of movies from  $p_1 = 6$  to  $p_1 = 9$ . As a consequence, the movie theater decides to raise Susan's hourly wage to \$9 per hour, i.e.  $W = 9$ . Calculate how many movies and steaks she consumes. Discuss all the relevant effects of the wage and price changes on her decision.

$$\text{Now, } MRS = \frac{-p_1}{p_2} \Rightarrow -\frac{2x_2}{x_1} = -\frac{9}{3}$$

$$\Rightarrow 6x_2 = 9x_1 \Rightarrow x_2 = \frac{3}{2}x_1$$

Her new budget constraint is

$$9x_1 + 3x_2 = 9k$$

$$\text{So } 9x_1 + \frac{3}{2}x_1 = 9k$$

$$\Rightarrow x_1 = \frac{2}{3} \cdot 9k = 8$$

$$9 \cdot \frac{2}{3}x_2 + 3x_2 = 9k \Rightarrow x_2 = k = 12$$

So her new bundle is 8 movies and 12 steaks.

Her original utility was  $U(20/3, 20/3) = e^{(20/3)^2 \cdot 20/3}$

To calculate the substitution effect, we find the point

on this indifference curve with  $MRS = -\frac{p_1'}{p_2} = -\frac{9}{3}$

Recall that  $MRS = -\frac{2x_2}{x_1}$ , so  $x_2 = \frac{3}{2}x_1$ .

$$e^{(x_1)^2 \cdot \frac{3}{2}x_1} = e^{(20/3)^2 \cdot 20/3} \Rightarrow \frac{3}{2}x_1^3 = (20/3)^3$$

$$\Rightarrow x_1 = \sqrt[3]{\frac{2}{3} \cdot \frac{20}{3}}, \quad x_2 = \frac{3}{2} \sqrt[3]{\frac{2}{3} \cdot \frac{20}{3}}$$

$$x_1 \approx 5.824, \quad x_2 \approx 8.736$$

10 The substitution effect causes her consumption to shift from  $(20/3, 20/3) \approx (6.667, 6.667)$  to  $(5.824, 8.736)$ , and the income effect moves her consumption from there to  $(8, 12)$ .

from  $p$  change? from  $w$  change?

