Massachusetts Institute of Technology
Department of Economics

14.01 Principles of Microeconomics
Midterm #2
Tuesday, November 2, 2004

Please print legibly.

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Student ID: 968 819 927

IMPORTANT: Please circle your section.

Edward Cho		10 AM			1 PM	2 PM
Saugato Datta			11 AM		1 PM	
Alan Grant		10 AM	11 AM	12 PM		
Ummul Hasanath Ruthbah	9 AM					2 PM

Instructions:

You have two hours to complete this exam. The test has a total of 100 points on 12 pages. You should try to complete each section in 30 minutes leaving 30 minutes at the end to check your answers. You may use a non-programmable calculator. We encourage you to write your answers in pen as it facilitates any regrading requests you may have.

Do not write anything in the area below.

Question 1:	5 /5	Question 6(a):	8 /8	Question 7(a):	7 /7
Question 2:	5 /5	Question 6(b):	6 /6	Question 7(b):	7 /7
Question 3:	5 /5	Question 6(c):	7 /7	Question 7(c):	4 /6
Question 4:	5 /5	Question 6(d):	8 /8	Question 7(d):	6 /6
Question 5:	2 /5	Question 6(e):	7 /8	Question 7(e):	6 /6
Question 6:	36 /37			Question 7(f):	6 /6
Question 7:	36 /38				
TOTAL:	94 /100				

Section I: True/False/Uncertain

In this section, write whether each statement is true, false, or uncertain. Please fully explain your answer, using a diagram if appropriate. No credit will be given for an answer without an explanation.

5

1. (5 points) Cost minimization implies profit maximization.

False. Cost minimization is a necessary component for profit maximization, but is not sufficient. Cost minimization is necessary to choose the appropriate input bundle to produce a given quantity, but it does not specify the amount to be produced. For this, we must set $\frac{\partial \pi}{\partial p} = 0$ $\Rightarrow MR = MC$.

5

2. (5 points) Redistributing the profits of a monopolist to consumers eliminates the deadweight loss of monopoly.

False. The deadweight loss is the difference between the total of consumer surplus and producer surplus at a given price/quantity outcome and that at equilibrium. Redistributing profits to consumers shifts the producer surplus to consumers. It doesn't change the total surplus, and so DWL is unchanged.

3. (5 points) A profit maximizing firm is currently using labor and capital in the ratio 1:2. If the price of labor goes up, while the price of capital remains the same as before, the firm will change its optimal input mix.

5

Not necessarily. If, e.g. the production function is the Leontief production function $q = \min(2L, K)$, labor and capital will always be consumed at a ratio of 1:2 in the optimal input mix.

5

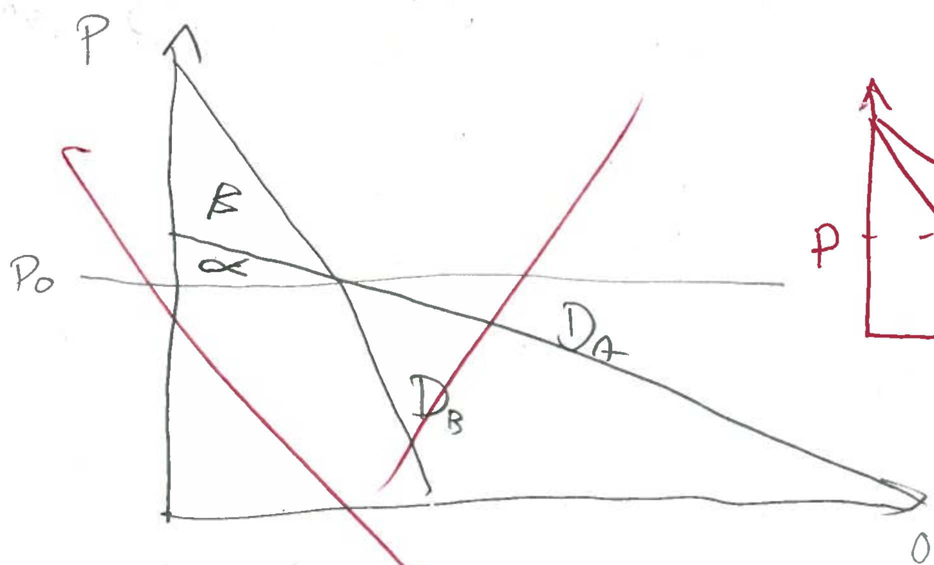
4. (5 points) A perfectly competitive firm is operating at a point where its MC = \$4.5 and its AC = \$3. If it were to increase its output slightly, its AC would rise.

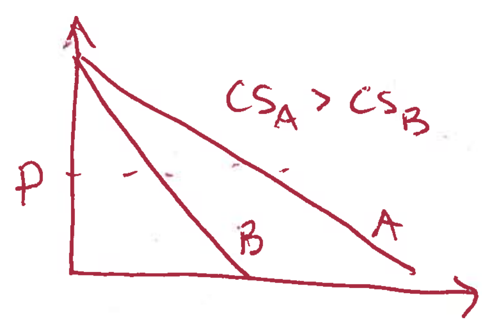
True. $AC = \frac{TC}{q}$, $MC = \frac{dTC}{dq}$. When $MC > AC$, the next unit produced costs more than the average cost of previous units, raising AC.

2

5. (5 points) Good A and Good B both have linear demand curves, but the slope of Good A's demand curve is lower in absolute value than the slope of B's demand curve. Both sell at the same price. It must be the case that consumer surplus is higher in the case of Good B than in the case of Good A.

$\boxed{\text{False.}}$ Consider the counterexample:





The slope of D_A is less than D_B, but A has a CS of only α while B has a surplus of α + β.

Section II: Long Answer Question #1

This question is in parts and is cumulative. If you cannot solve part of the question and you need the results of that part in later parts, you should assume a generic functional form (*e.g.* $q_S(p) = S(p)$) or describe what you would do with the solution from the earlier part to receive partial credit.

6. (a) (8 points) A perfectly competitive industry for rice has many identical farmers, who produce rice according to the production function

$$q = S^{1/2}L^{1/2}$$

where q is the output of rice in pounds, L is units of labor, and S is units of the other input into production, seeds. The price of labor is \$2 and the price of seeds is \$4 per unit. In the short run, however, the amount of seeds farmers use is fixed at $S = 16$, so that a farmer can only vary labor. Derive the farmer's short run total cost, average cost, average variable cost and marginal cost functions.

S is fixed, so

$$q = \bar{S}^{1/2} L^{1/2} = 4 \cdot L^{1/2} \Rightarrow L(q) = \left(\frac{q}{4}\right)^2 = \frac{q^2}{16}$$

$$TC(q) = wL + r\bar{S} = 2L(q) + 4\bar{S}$$

$$= \frac{2q^2}{16} + 4 \cdot 16 = \frac{q^2}{8} + 64$$

$$AC = \frac{1}{q}\left(\frac{q^2}{8} + 64\right) = \frac{q}{8} + 64/q$$

$$AVC = q/8$$

$$MC = \frac{dTC}{dq} = \frac{2q}{8} = q/4$$

8

(b) (6 points) Suppose that the price of rice is \$8 a pound. How many pounds of rice would a profit-maximizing farmer produce in the short run? Depict the farmer's output decision on a diagram. Calculate the profit that the farmer makes.

$MR = P = 8$

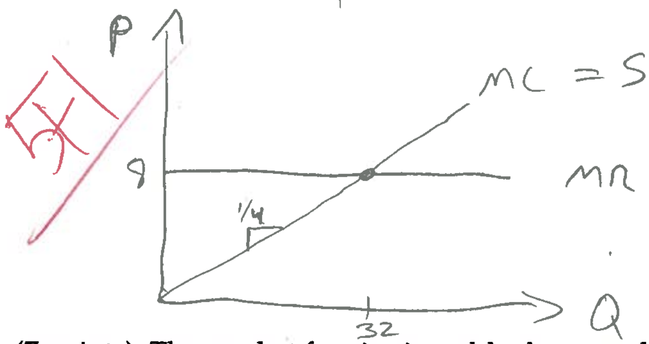
At max profits, $MR = MC \Rightarrow 8 = q/4$

$\Rightarrow \boxed{q = 32}$ ✓

$\pi = pq - C(q) = 32 \cdot 8 - C(32)$

$= 256 - \left(\frac{32^2}{8} + 64\right) = 256 - 192 = \boxed{64}$

check for AVC



$MC = q/4 > AC = q/8$ $\forall q \in \mathbb{R}^+$, so the firm will never shutdown

(c) (7 points) The market for rice is suddenly overtaken by a fad diet which discourages the consumption of carbohydrates which rice is rich in. Demand for rice drops drastically, leading to a decline in its market price to \$3. Find the short-run output and profit of the representative farmer and explain your answer.

$MR = P = 3$

We still have $MR = MC$, so $3 = q/4$

$\Rightarrow \boxed{q = 12}$ ✓ Still no shutdown (see above).

$\pi = Pq - C(q) = 12 \cdot 3 - C(3) =$

$= 36 - \left(\frac{3^2}{8} + 64\right) = 36 - 68$

6 ✓ $\boxed{= -32}$

The firm is now losing money, but the loss is less than the fixed costs of 64 that would be incurred even if it produced nothing. So it continues to operate.

(d) (8 points) Supposing we are now in the long run and the anti-carbohydrate diet still holds sway, so that the price of rice is still \$3. How would a rice-producing farmer combine labor and seeds (the amount of which he can now vary) to produce the number of units of rice he was producing in the short run (If you couldn't find the number of units he was producing in the short run, call it q)? Derive the firm's LRAC and LRMC functions. Would the farmer indeed produce as many units of rice in the long run as he did in the short run? If not, how much would the farmer produce instead? Explain. (Note: Continue to use the production function and w and r from part (a))

We can now vary the amount of seeds. To minimize costs,

$$MRTS = \frac{MP_L}{MP_S} = \frac{w}{r}$$

$$MP_L = S^{1/2} L^{-1/2} \cdot \frac{1}{2}, \quad MP_S = L^{1/2} S^{-1/2} \cdot 1/2$$

$$MRTS = \frac{S^{1/2} L^{-1/2} \cdot 1/2}{L^{1/2} S^{-1/2} \cdot 1/2} = \frac{S}{L} = \frac{w}{r} = \frac{2}{4}$$

$$4S = 2L \Rightarrow L = 2S.$$

So $q = S^{1/2} L^{1/2} = S^{1/2} (2S)^{1/2} = \sqrt{2}\, S$

$\Rightarrow S(q) = q/\sqrt{2}, \quad L(q) = 2S(q) = \sqrt{2}\, q$

To produce 12 lbs as in the SR, he chooses $\boxed{S = 12/\sqrt{2}, \quad L = \sqrt{2} \cdot 12}$

8

$C(q) = wL(q) + rS(q) = 2\sqrt{2}\, q + 4\, q/\sqrt{2} = 4\sqrt{2}\, q$

$$\boxed{\begin{aligned} LRAC(q) &= 4\sqrt{2}q/q = 4\sqrt{2} \\ LRMC(q) &= \frac{d}{dq}(4\sqrt{2}q) = 4\sqrt{2} \end{aligned}}$$

The LRMC is horizontal at $4\sqrt{2}$. So to maximize profits, the farmer will produce as much as possible (ideally an infinite amount) at all prices greater than $4\sqrt{2}$. At prices lower than $4\sqrt{2}$, such as the price of 3, he produces nothing.

(e) (8 points) The production of rice is now revolutionized by the discovery of much higher-yielding, genetically modified seeds. If a farmer uses these new seeds, his output is given by $q = 4S^{1/2}L^{1/2}$. The new seeds in addition cost exactly as much as the old seeds, and since they're easy to produce, there's no restriction on how many of the new seeds the farmer can use. Would this change the farmer's optimal labor:seed ratio? At what price would the rice market be in a long-run competitive equilibrium? What would producer surplus be in this long-run competitive equilibrium? Supposing that total market demand for rice was given by $Q^d = 100 - p^2$, can you determine how many rice farmers there would be in the long-run competitive equilibrium, and how much rice each of them produces?

There is no change in the optimal labor/seed ratio since the MRTS doesn't change and the new seeds have the same price. The farmer simply produces 4 times as much with the old seeds. So $C(q)$ decreases by a factor of 4. Now $C(q) = \sqrt{2}q$, and so $LRAC(q) = LRMC(q) = \sqrt{2}$. The LR market ~~demand~~ curve is horizontal at the minimum AC, which is simply $\sqrt{2}$. Producer surplus is zero as in all perfectly competitive equilibria. With this demand curve, we know $Q_d = 100 - (\sqrt{2})^2 = 98$ units are produced, but because the min AC does not occur at a fixed quantity, we can't tell how many firms are in the market.

7

Section III: Long Answer Question #2

This question is in parts and is cumulative. If you cannot solve part of the question and you need the results of that part in later parts, you should assume a generic functional form (*e.g.* $q_S(p) = S(p)$) or describe what you would do with the solution from the earlier part to receive partial credit.

7. Fairy's Dream is a monopoly firm producing dreams. The demand for dreams in the market is given by $Q = 100 - P$. The cost function for producing dreams is $C(Q) = 10Q$.

   (a) (7 points) Calculate the equilibrium quantity and price of dreams in the market. What are the consumer surplus, producer surplus, and deadweight loss (DWL)?

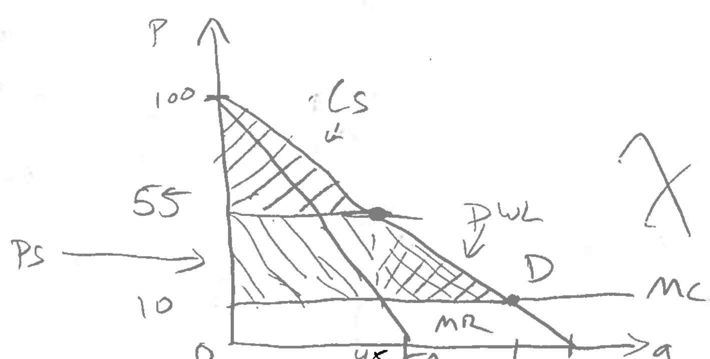
$p(q) = 100 - q$

$MR = p(q) + q\,p'(q) = 100 - q + 1 \cdot -1 = 100 - 2q$

$MC = 10$

$MR = MC \Rightarrow 100 - 2q = 10 \Rightarrow \boxed{q = 45}$

$p(45) = 100 - 45 = \boxed{55}$



X

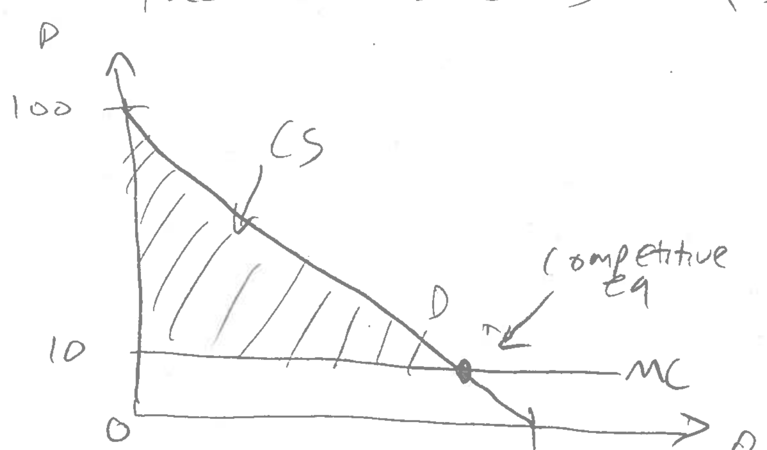
$CS = \frac{1}{2} \cdot 45 \cdot 45 = \frac{2025}{2}$

$PS = 45 \cdot 45 = 2025$

$DWL = \frac{1}{2} \cdot 45 \cdot 45 = \frac{2025}{2}$

   (b) (7 points) Suppose now that the government forces Fairy's Dreams to produce efficiently. In that case calculate consumer surplus, producer surplus, and DWL.

Then $MC = D \Rightarrow 10 = 100 - Q \Rightarrow Q = 90,\ P = 10$



$DWL = 0$ by def

$PS = 0$

$CS = \frac{1}{2} \cdot 90 \cdot 90 = 4050$

X

no PS

(c) (6 points) After some time selling dreams, a court finds that Fairy's Dream actually stole the formula for producing dreams from a chemist who graduated from MIT. Since dreams are good for the society and Fairy's Dream is an experienced producer the court decides to fine it rather than shutting it down. Fairy's Dream has two options. It can pay a fixed amount of \$450 or it can pay \$10 per unit produced to the chemist. Calculate the equilibrium price, quantity, consumer surplus, producer surplus, and DWL under these two options.

Option 1 - fixed amount.

$C(q)$ is now $10Q + 450$.

$MC = 10$, just as before. And MR & demand give the same, so we get the same outcome except that the monopolist must pay the chemist, so

$P = 55$, $q = 45$, $CS = 2025/2$, $PS = \cancel{2025}$ $-450 = 1575$

& Chemist's receipts $= 450$, DWL $= 2025/2$

Option 2 - \$10/unit

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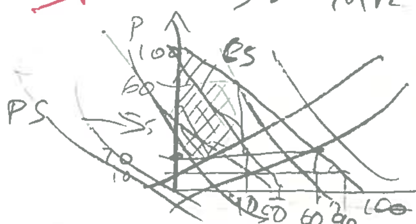
$C(q) = 10q + 10q = 20q \Rightarrow MC = 20$, MR is still $100 - 2q$,

so $MR = MC \Rightarrow 20 = 100 - 2q \Rightarrow q = 40$, $p = 60$.



$CS = \frac{1}{2} \cdot 40 \cdot 40 = 800$

$PS = 40 \cdot 40 = 1600$

Chemist's receipt $= 10 \cdot 40 = 400$

(over)

(d) (6 points) What is the efficient type of fine for society? Why? Which one is preferred by the monopolist? Why? What does the chemist prefer? Why?

The monopolist prefers the per-unit fee because it makes PS = \$1600 rather than \$1575.

The chemist prefers the flat fee because he makes \$450 instead of \$400.

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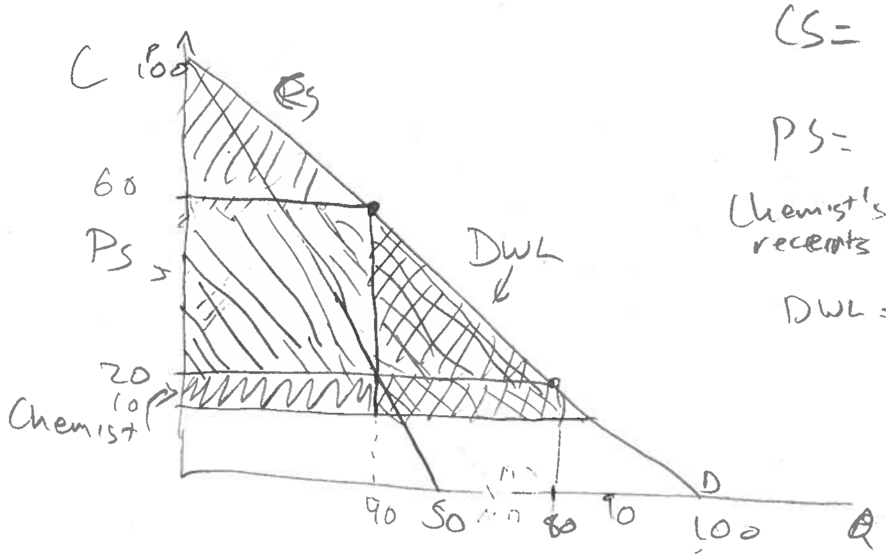
And the flat fee is more efficient to society because DWL = 1012.5 < 1250.



$CS = \frac{40 \cdot 40}{2} = 800$ ✓

$PS = 40 \cdot 40 = 1600$ ✓

Chemist's receipts $= 10 \cdot 40 = 400$

$DWL = \frac{50 \times 50}{2} + = \cancel{1250}$

$\frac{40 \times 40}{2}$

(e) (6 points) Instead of the situation in part (c), suppose the monopoly firm does not have any marketing skill. So it gives another firm, All Your Dreams (AYD), the sole right to market its dreams. Fairy's Dream first fixes a price (P_r) at which it will supply dreams to AYD and AYD then sets its monopoly price (P_f) for dreams. AYD does not have any other cost for marketing dreams. What is the price (P_r) charged by Fairy's Dreams? What is the price (P_f) charged by AYD?

The quantity demanded is still $Q = 100 - P_f \Rightarrow P_f = 100 - Q$

So $MR_{AYD} = P_f(q) + P_f'(q)\,q = 100 - 2q$

The cost to AYD is $C_{AYD}(q) = P_r q \Rightarrow MC_{FD} = P_r$

So $MC_{AYD} = MR_{AYD} \Rightarrow 100 - 2q = P_r \Rightarrow q = 50 - \frac{P_r}{2}$

And for FD,

$R_{FD} = PQ \Rightarrow MR_{FD} = P_r(Q) + P_r'(Q)\,Q$

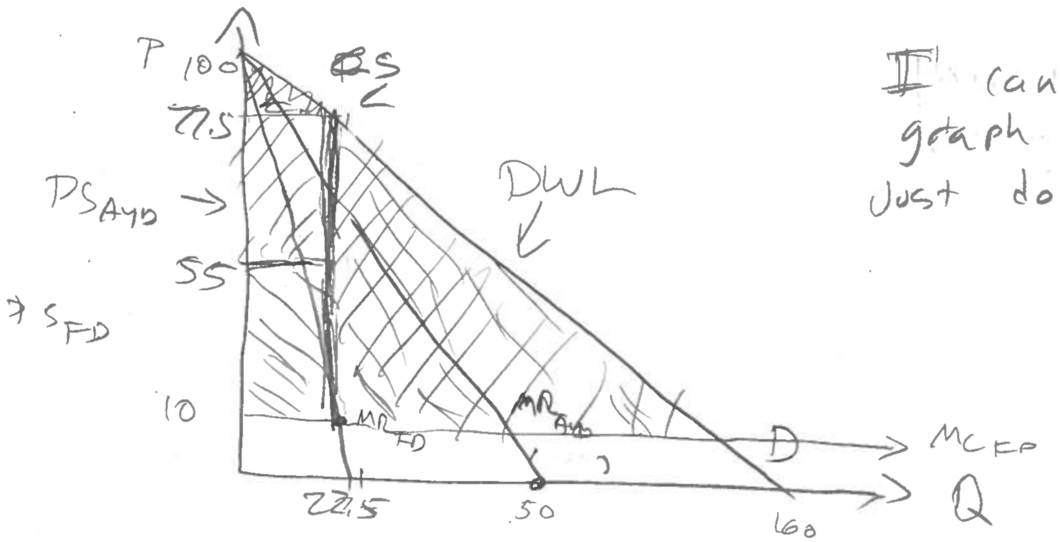
$MR_{FD} = 100 - 2q + -2q = 100 - 4q$

$MR_{FD} = MC_{FD} \Rightarrow 100 - 4q = 10$ since MC_{FD} is still 10

$$\boxed{\begin{aligned} q &= 90/4 = 22.5 \\ P_r &= 100 - 2q = 55 \\ P_f &= 100 - q = 77.5 \end{aligned}}$$

6

(f) (6 points) Calculate consumer surplus, producer surplus, and DWL when there are two monopolies (one producing dreams and the other marketing them) in the market for dreams. Why is social welfare differ in this situation as compared to the situation in parts (a) and (b)? Which outcome is socially preferable?



I can't read this graph either, I'll just do the integrals,

The consumer surplus is $\int_0^{Q_e} D^{-1}(q) - P_f \, dq$

$= \int_0^{22.5} 100 - q - 77.5 \, dq = \boxed{253.125}$

The producer surplus for AYD is $\int_0^{Q_e} P_f - MC_{AYD}(q) \, dq$

$= \int_0^{22.5} 77.5 - P_r \, dq = \int_0^{22.5} 77.5 - 55 \, dq$

$= 506.25$

The producer surplus for FD is $\int_0^{Q_e} P_r - MC_{FD}(q) \, dq$

$= \int_0^{22.5} 55 - 10 \, dq = 1012.5$

So the total producer surplus is $506.25 + 1012.5$

$= \boxed{1518.75}$

And the DWL is $4050 - 1518.75 - 253.125 \approx \boxed{2278.13}$

Since 4050 was the total surplus at competitive equilibrium

6

(over)

Social welfare is reduced because there are two monopolists setting their prices to maximize their own profits. The DWL is much greater than before, so the single monopolist is socially preferable.