Name: DAN PORTS

M.I.T. ID# 968 819 927 Section: EF

15.401 Midterm Examination

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Fall 2011

Please make sure that your copy of the exam contains 28 pages (including this one). Write your name and MIT ID number on *every* page.

- You are allowed one $8\frac{1}{2}$"$\times$11" sheet of notes and one non-programmable non-PDA calculator.
- Answer these questions *without* consulting anyone.
- **Even after you hand in this examination, do not discuss it with anyone until all sections have finished taking it (4:00PM EST). If you discuss any part of the examination with any other student prior to 4:00pm EST, this will be considered an act of cheating by both parties and will be prosecuted accordingly.**
- You have eighty (80) minutes to complete this examination. Credit for each question is proportional to the amount of time you ought to spend on it. Therefore, do not agonize over a 5-point question without having tackled a 30-point question.
- Use the space provided. If more space is needed, use the other side of the page.
- Be neat and show your work. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.
- For simplicity, assume that the coupon payments for all coupon bonds occur annually, i.e., once a year, not semi-annually as is the convention in practice.

Good luck!

Some Useful Formulas

$$\text{PV(Annuity)} = \sum_{k=1}^{n} \frac{C}{(1+r)^k} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^n}\right] \quad (1)$$

$$\text{PV(Perpetuity)} = \sum_{k=1}^{\infty} \frac{C}{(1+r)^k} = \frac{C}{r} \quad (2)$$

$$\text{DDM with no growth: Price} = \sum_{k=1}^{\infty} \frac{D}{(1+r)^k} = \frac{D}{r} \quad (3)$$

$$\text{DDM with constant growth: Price} = \sum_{k=1}^{\infty} \frac{D(1+g)^{k-1}}{(1+r)^k} = \frac{D}{r-g} \quad (4)$$

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15.401 Midterm Examination 2011 Grade Sheet

1.	10	/	10
2.	15	/	15
3.	20	/	20
4.	21	/	25
5.	25	/	25
6.	25	/	25
Total	116	/	120

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Question 1 (10 points): 5

True or false? Briefly explain (or qualify) your answers.

(a). (5 points) Consider two riskless (no-default) bonds, A and B. Suppose that bond A has a higher yield-to-maturity than bond B. Then, rational investors should buy bond A and sell bond B until their yield-to-maturities are equalized.

✓ FALSE. The two bonds may have different maturity dates, in which case we'd expect their YTM to vary according to the term structure of interest rates.

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Question 1 (continued) 5

(b). (5 points) If the yield to maturity increases, all else equal, then the duration of a coupon bond decreases.

✓ TRUE.

$$D = \sum_{t=1}^{T} \frac{1}{P} t \frac{Ct}{(1+y)^t}$$

As y increases, the weighting of the later terms in the series decreases, causing the bond's duration to decrease.

(The bond's present value P also decreases, but the factor above is dominant.)

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Question 2 (15 points)

Cute Lizard Insurance has a portfolio containing two coupon-paying bonds, bond A and bond B. Bond A has a market value of \$25 million and a duration of 2 years. Bond B has a market value of \$50 million and a duration of 5 years. The term structure of interest rates is constant at 5% per year for all maturities.

The question consists of two parts.

5 (a). (5 points) Calculate the duration of the portfolio.

$$D = \frac{1}{\$75M}\left(2 \cdot \$25M + 5 \cdot \$50M\right)$$

$$= \boxed{4 \text{ years}}$$

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Question 2 (continued)

10 (b). (10 points) Suppose that the interest rates for all maturities increase by the same amount (i.e. from 5% to r>5%), and this causes the portfolio of bonds to lose $2 million in value. What is the approximate change of the interest rates?

$$MD = \frac{D}{1+y} = \frac{4}{1.05} \approx 3.810$$

$$\Delta P = -P \cdot MD \cdot \Delta y$$

$$-\$2M = -\$75M \cdot 3.810 \cdot \Delta y$$

$$\Delta y = \frac{\$2M}{\$75M \cdot 3.810}$$

$$\approx 0.007 = 0.7\%$$

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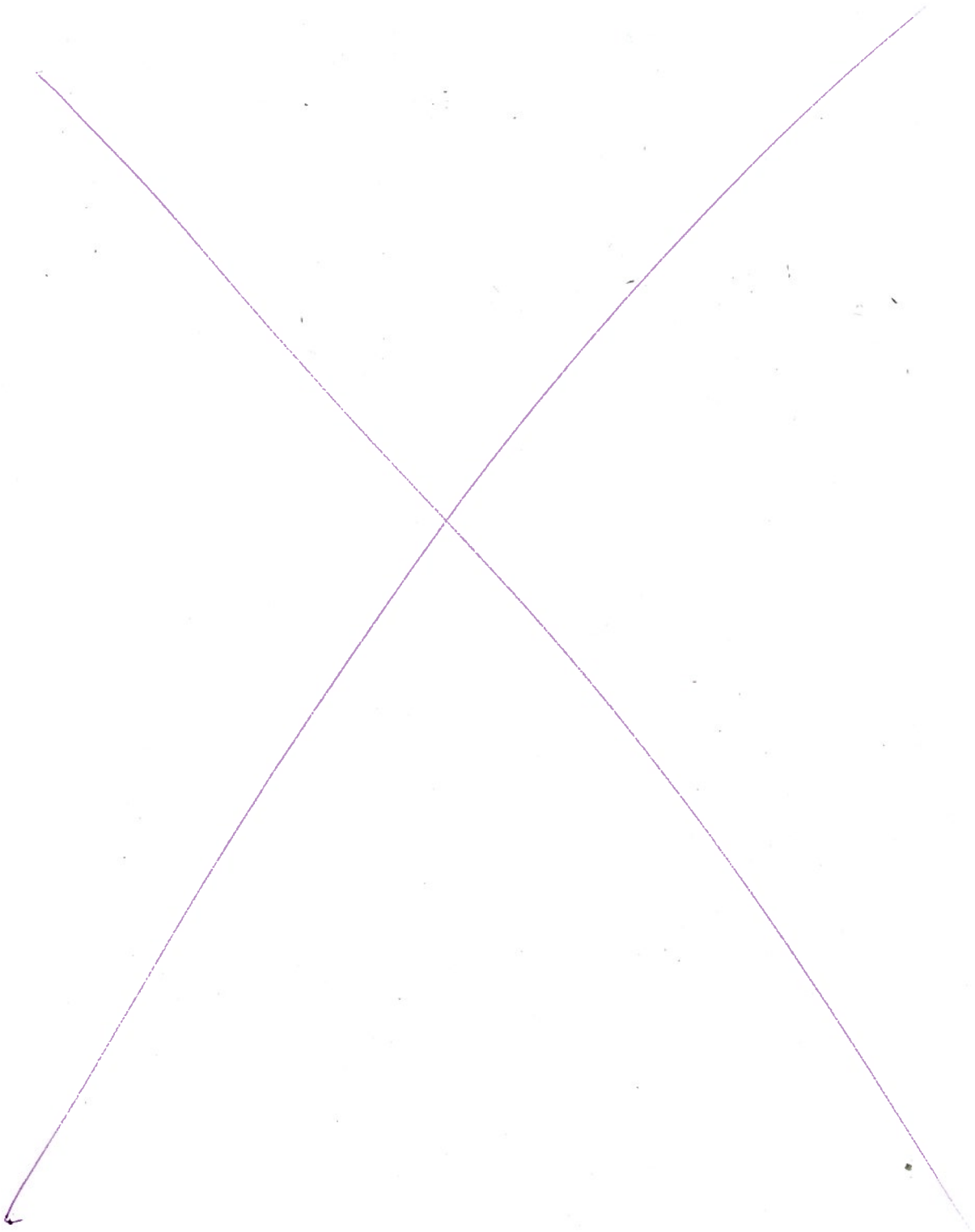
Extra Space for Question 2



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Question 3 (20 points) 10

This question has two parts.

(a). A bank offers a savings account with an *annual percentage rate* (APR) of 11%. What is the effective annual interest rate (EAR) offered by the bank if the compounding interval is:

- (5 points) Monthly?

$$EAR = \left(1 + \frac{0.11}{12}\right)^{12} - 1$$

$$\approx \boxed{11.57\%}$$

- (5 points) Weekly? Assume 52 weeks in a year.

$$EAR = \left(1 + \frac{0.11}{52}\right)^{52} - 1$$

$$\approx \boxed{11.61\%}$$

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Extra Space for Question 3a

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Question 3 (continued)

(b). (10 points) You have the possibility to buy a 2010 Ferrari California for $200,000 (a very good deal!). You have no initial savings, but with the fabulous job you get after graduation you are able to save $3,900 per month. The bank offers you a 5-year loan at 6% APR, compounded monthly. Are your savings of $3,900 per month enough to afford the monthly payment on the car?

6% APR $\Rightarrow$ 0.5% /mo

5 yr $\Rightarrow$ 60 m

$$\$\,200{,}000 = \frac{A}{0.005}\left(1 - \frac{1}{(1.005)^{60}}\right)$$

$\Rightarrow A = \$3866.56$

monthly payment

Yes, barely. ✓

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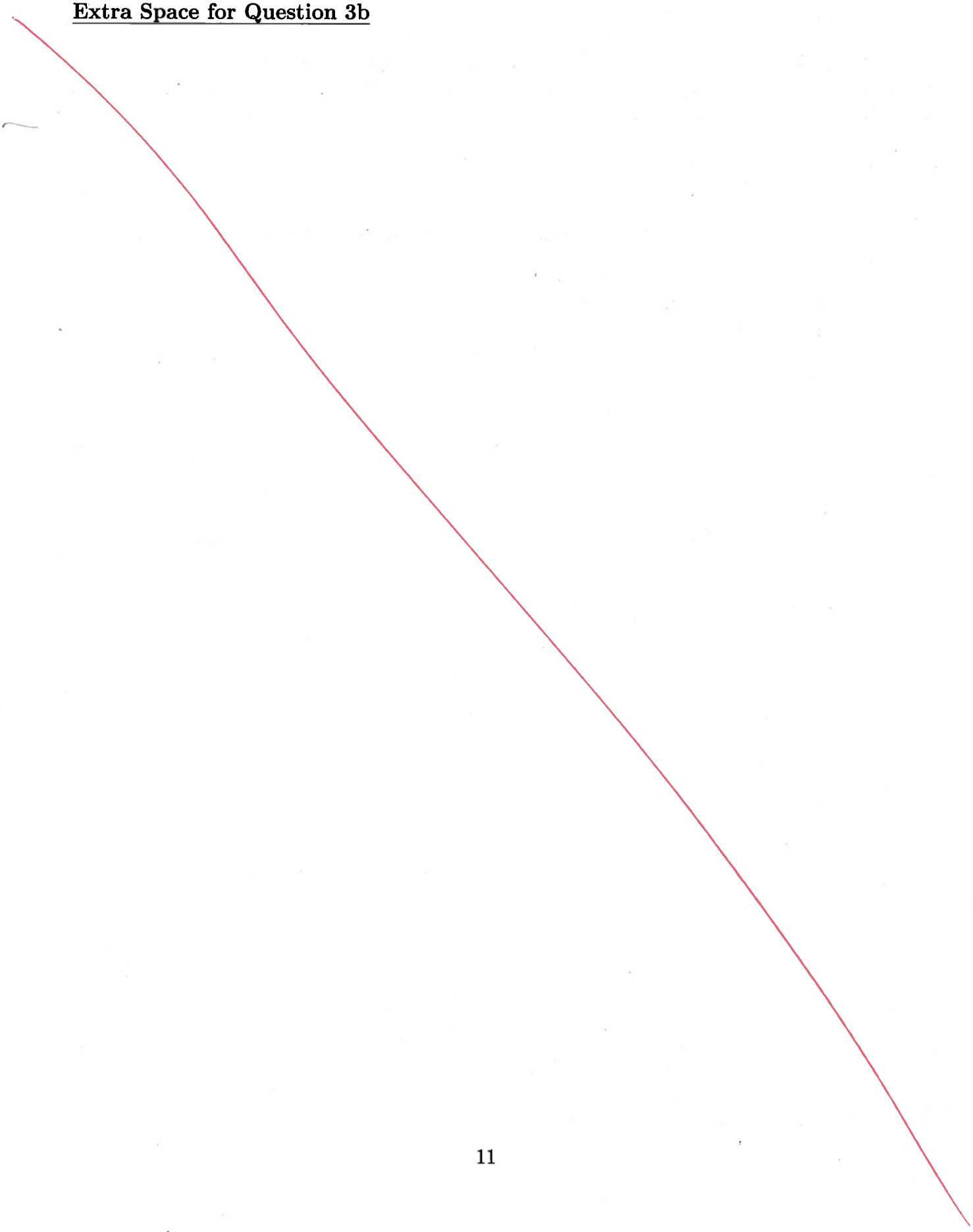
Extra Space for Question 3b



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Question 4 (25 points)

The current price of silver is \$35.00 per ounce. The storage costs are \$0.10 per ounce per month payable monthly at the beginning of each month. Assume that the convenience yield of holding silver is zero. Investors can borrow and lend at 6% APR, compounded monthly (0.5% per month).

This question has two parts.

(a). (10 points) Using the no arbitrage principle, determine the futures price of one ounce of silver for delivery in three months.

$$\hat{y} = -\frac{\$0.10}{\$35.00} \approx .286\%$$

$$F_{0,3} = S_0(1 + r_f - \hat{y})^3$$

$$= \$35(1 + 0.005 - 0.00286)^3$$

$$\approx \$35.83$$

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Extra Space for Question 4a

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Question 4 (continued)

(b). (15 points) Suppose that the actual futures price of silver for delivery in three months is \$36.00 per ounce. One futures contract of silver is written on 5,000 ounces. How would you construct a risk-free strategy to make money (arbitrage)? To get full credit, say *precisely* what you would buy or sell, and how much money you would borrow or deposit into a bank account and for how long.

Futures are overpriced ⇒ sell them short.

Strategy:
- sell short 1 3 mo contract
- buy & store until $t=3$ 5000 oz silver at $t=0$
- borrow \$175,500 from $t=0$ to $t=3$
- borrow \$500 from $t=1$ to $t=3$
- borrow \$500 from $t=2$ to $t=3$

Cash flows:

	$t=0$	$t=1$	$t=2$	$t=3$
- short future				\$36.5000 = \$180,000
- buy silver	-\$35.5000 = -\$175,000			
- storage	-\$1.5000 = -\$500	-\$500	-\$500	
- borrow \$175,500 from $t=0 \to 3$	\$175,500			$1.005^3 \cdot \$175,500$ = -\$178145.68
- borrow \$500 from $t=1 \to 3$		\$500		$1.005^2 \cdot \$500$ = -\$505.01
- borrow \$500 from $t=2 \to 3$			\$500	$1.005 \cdot \$500$ = -\$502.5
net:	0	0	0	⇒ \$846.80

FV(net profit)

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Extra Space for Question 4b

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Question 5 (25 points)

Dreams Inc. has a cost of capital of 10% per year. Its expected *earnings per share* (EPS) next year are \$2.00. The firm plans to pay out 40% of its earnings as dividends and plow back the remaining 60% of its earnings for new investments in the following years. The *return on equity* (ROE) on the new investments is 15%.

The question consists of three parts.

10

(a). (10 points) Calculate the share price and the (forward) *price to earnings ratio* (P/E) of this firm? The forward P/E ratio is defined as the share price today divided by the expected earnings per share over the next year, P_0/EPS_1.

$g = b \cdot ROE = 60\% \cdot 15\% = 9\%$

$D_1 = EPS_1 \cdot b = \$2 \cdot 0.4 = \$0.80$

$$P_0 = \frac{D_1}{r-g} = \frac{\$0.80}{0.1 - 0.09}$$

$= \boxed{\$80}$ ✓

$$P/E = \frac{P_0}{EPS_1} = \frac{\$80}{\$2} = \boxed{40}$$ ✓

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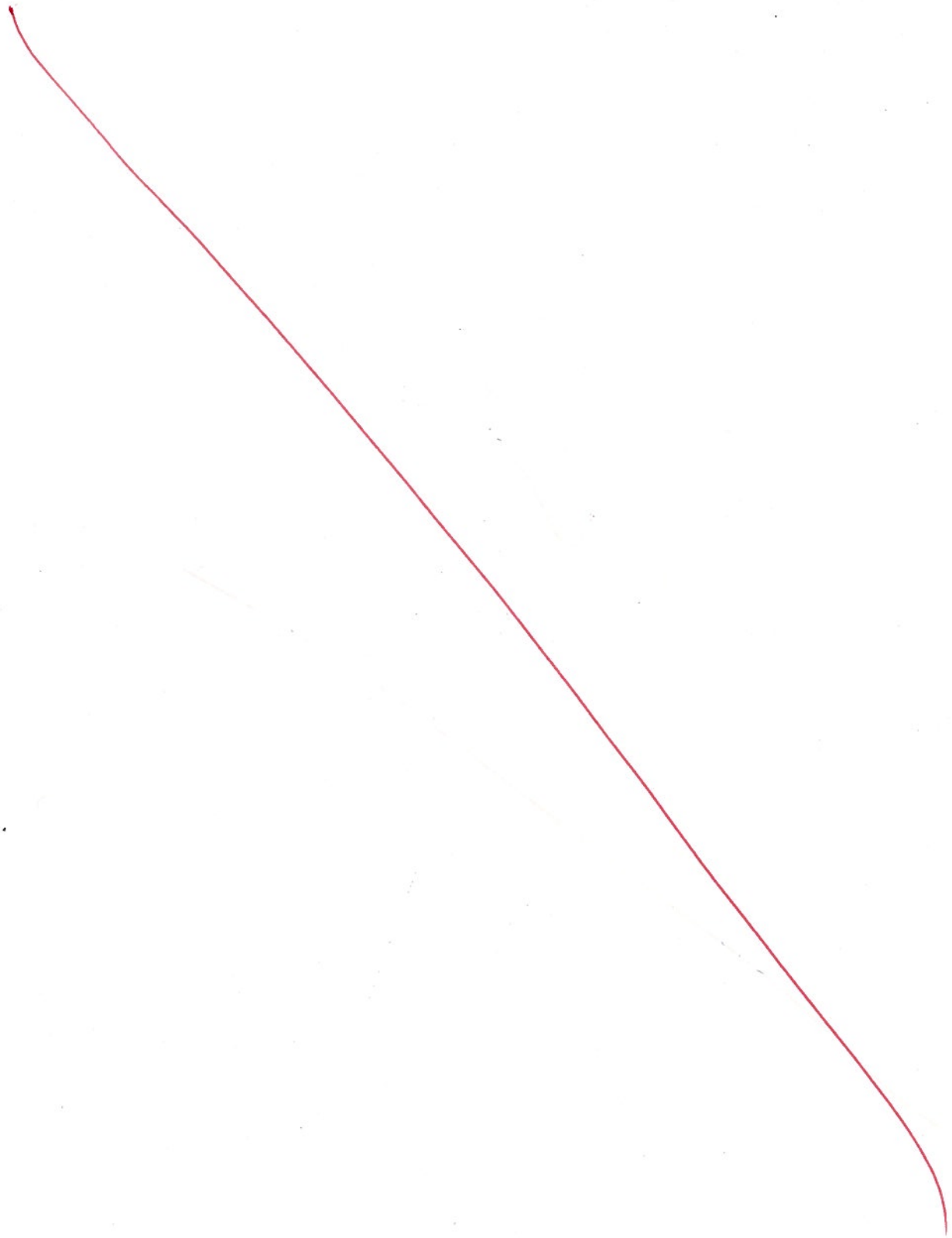
Extra Space for Question 5a



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Question 5 (continued)

(b). (5 points) How much of the share price is attributable to growth opportunities (i.e calculate the *present value of growth opportunities* or the PVGO)?

$$PVGO = P_0 - \hat{P}_0$$

$$\hat{P}_0 = \frac{EPS_1}{r} = \frac{\$2}{0.1} = \$20$$

$$PVGO = P_0 - \hat{P}_0 = \$80 - \$20$$

$$= \$60$$

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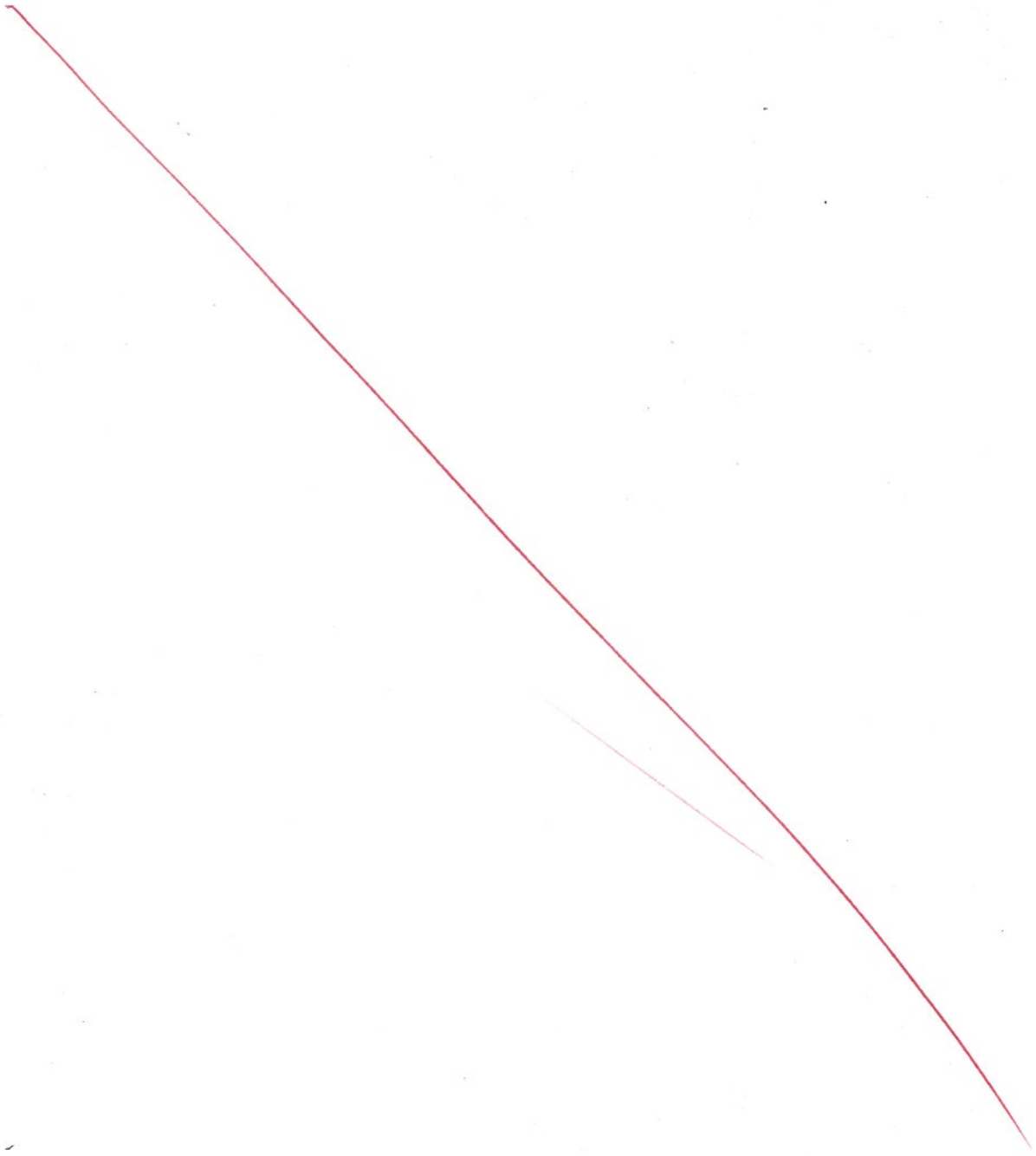
Extra Space for Question 5b



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Question 5 (continued) 10

(c). (10 points) If the payout ratio increases, will the P/E ratio increase, decrease, or remain unaffected? Assume that the return on the new investments remains unchanged.

DECREASE. ✓

Increase in payout ratio ⇒ decrease in plowback.

ROE > cost of capital ✓

so reinvesting earnings creates value and this is reflected in the price.

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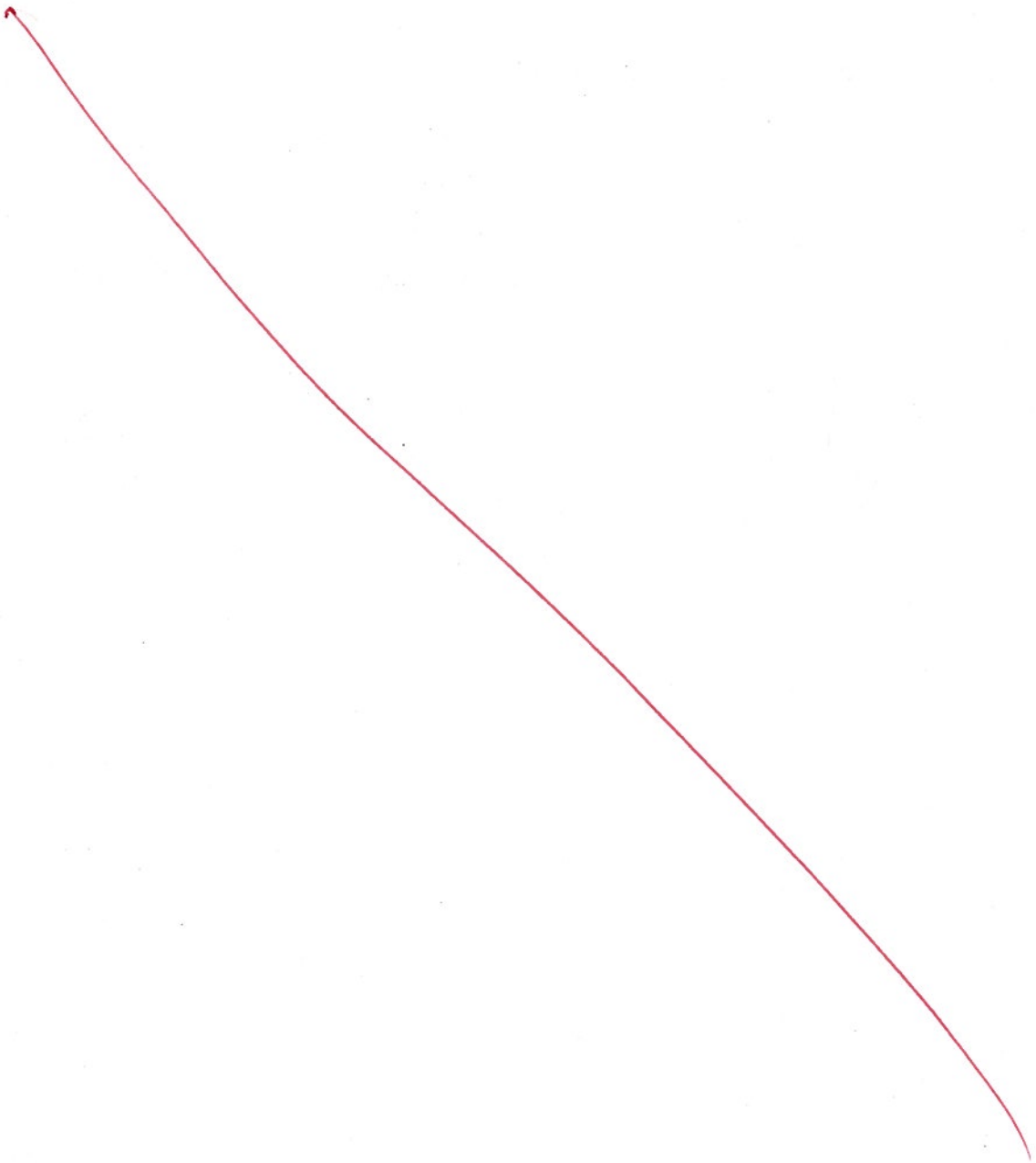
Extra Space for Question 5c



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Question 6 (25 points)

The table below gives the prices of riskless zero-coupon bonds that pay \$1,000 at maturity. For example, the price today of a security that pays \$1,000 for sure in two years is \$920.

Maturity (in years)	Zero-coupon bond price
1	\$970
2	\$920
3	\$875
4	\$830

Assume that there are no arbitrage opportunities in bond markets and ignore transaction costs. This question has two parts.

(a). (10 points) A riskless coupon bond with a \$1,000 face value and *annual* coupon payments is also trading in the bond market. Its coupon rate is 5% per year, and it has three years left to maturity (i.e., it will make three more coupon payments, in exactly one, two, and three years, and repay its face value in exactly three years). Determine its current price.

Can replicate w/ zeros to get price

year	cash flow	equiv zero price
1	\$50	\$50 · .97 = \$48.5
2	\$50	\$50 · .92 = \$46
3	\$1050	\$1050 · .875 = \$918.75

P = \$1013.25

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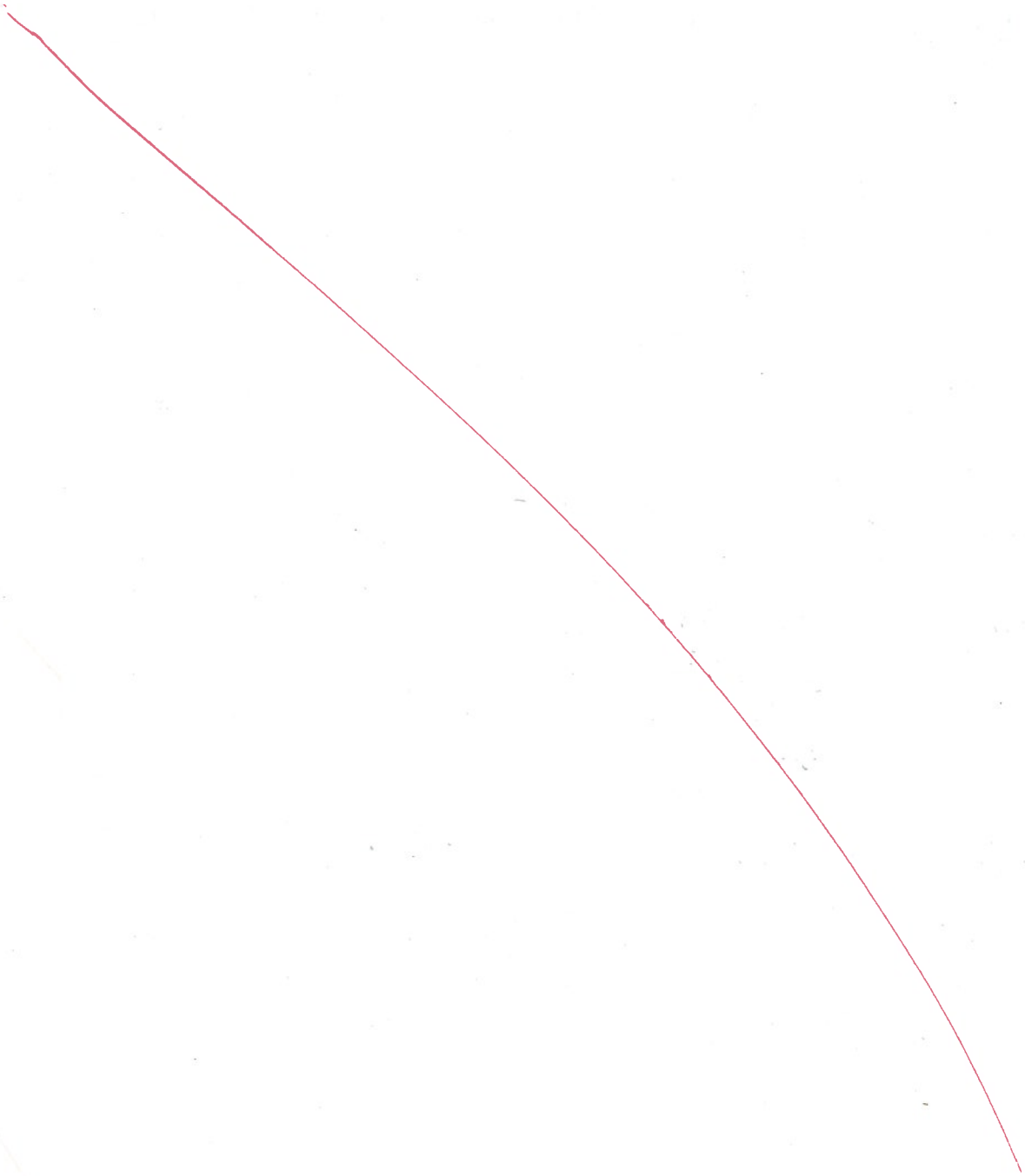
Extra Space for Question 6a



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Question 6 (continued)

(b). (15 points) A second riskless coupon bond with a \$1,000 face value and annual coupon payments is trading in the bond market. This bond has only two years left to maturity (i.e., it will make two more coupon payments, in exactly one and two years, and repay its face value in exactly two years), and its current yield to maturity is 4.2% (EAR). Can you determine the bond's coupon rate? If not, state what additional information you would need to determine the bond's coupon rate.

Spot rates inferred from zero prices:

$r_1 = \frac{\$1000}{\$970} - 1 = 3.09\%$

$r_2 = \left(\frac{\$1000}{\$920}\right)^{1/2} - 1 = 4.26\%$

price using spot rates = price using YTM

$\$1000\left(\frac{c}{1+r_1} + \frac{1+c}{(1+r_2)^2}\right) = \$1000\left(\frac{c}{1+y} + \frac{1+c}{(1+y)^2}\right)$

$\Rightarrow \frac{c}{1.0309} + \frac{1+c}{1.0870} = \frac{c}{1.042} + \frac{1+c}{1.0422}$

$c \cdot 1.1802 + (1+c)\,1.1193 = c \cdot 1.1677 + (1+c)\,1.1206$

$c(1.1802 + 1.1193) + 1.1193 = c(1.1677 + 1.1206) + 1.1206$

$c(1.1802 + 1.1193 - 1.1677 - 1.1206) = 1.1206 - 1.1193$

$0.0112\,c = 0.0013$

$c = 0.1161 = \boxed{11.61\%}$

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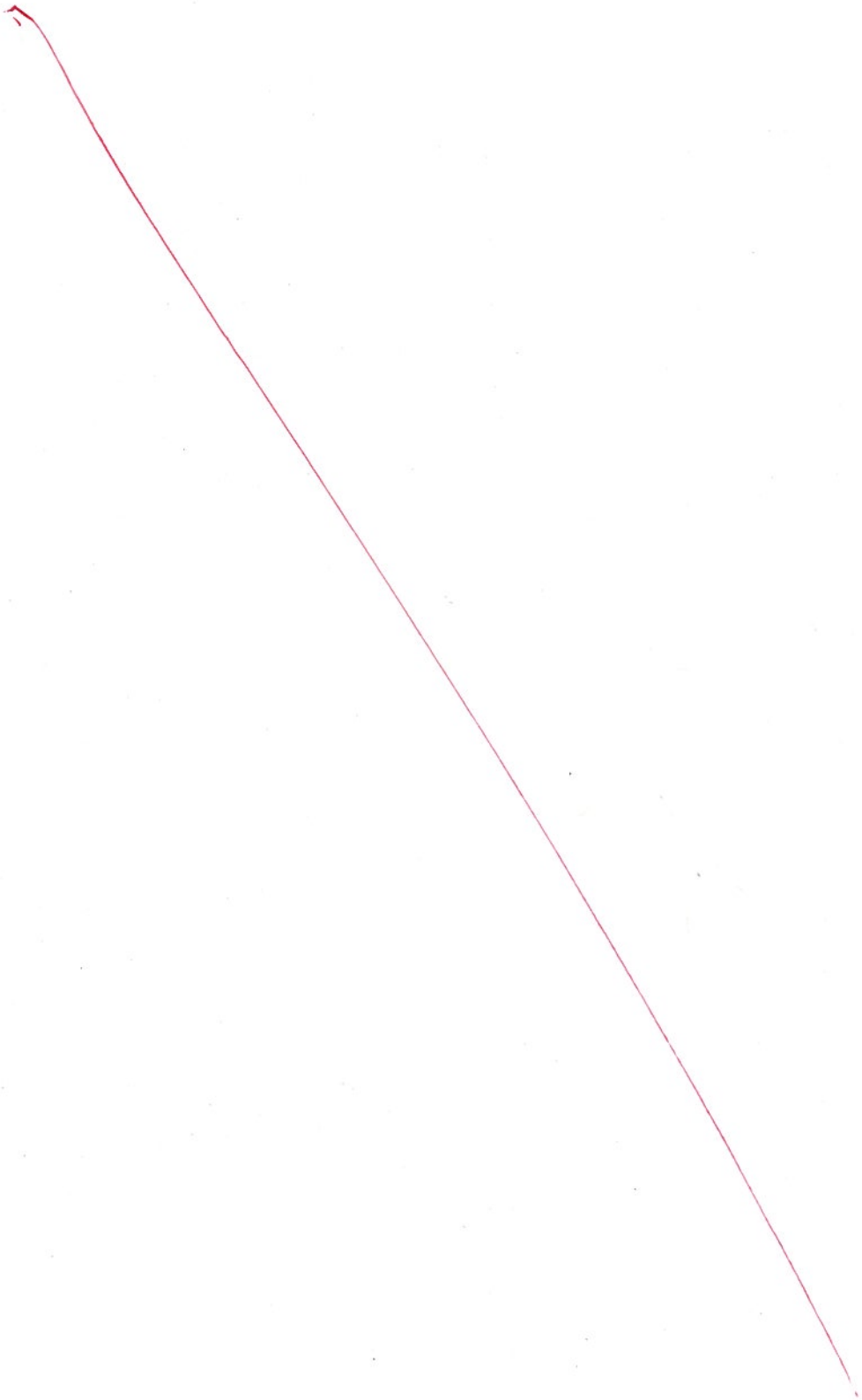
Extra Space for Question 6b



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