



Problem Set 2

Problem 1:

1) The T-bill has a six-month interest rate of $\frac{\$100000}{\$98058} - 1 \approx 1.98\%$, so its effective annual rate is $1.0198^2 - 1 \approx 4.00\%$.

The coupon bond pays 2.1% every six months, so its effective annual rate is $1.0021^2 - 1 = 4.2441\%$.
The coupon bond has the higher EAR.

2) With a YTM of 8%, the three-month interest rate is $1.08^{1/4} - 1 \approx 1.94\%$. The sale price of the bond, therefore, will be $\frac{\$100000}{1.0194} \approx \98094 . The holding period return is \$36.

Problem 2:

1)

$$\begin{aligned} PV_A &= \frac{\$160}{1.12} + \frac{\$160}{1.12^2} + \frac{\$1160}{1.12^3} && \approx \$1096.07 \\ PV_B &= \frac{\$40}{1.12} + \frac{\$40}{1.12^2} + \frac{\$1040}{1.12^3} && \approx \$807.85 \\ PV_C &= \frac{\$1000}{1.12^3} && \approx \$711.78 \end{aligned}$$

2) The Macaulay durations (not modified durations) of the three bonds are:

$$\begin{aligned} D_A &= \frac{1}{PV_A} \left(\sum_{i=1}^3 i PV_i \right) = \frac{1}{\$1096.07} \left(1 \cdot \frac{\$160}{1.12} + 2 \cdot \frac{\$160}{1.12^2} + 3 \cdot \frac{\$1160}{1.12^3} \right) \approx 2.623 \\ D_B &= \frac{1}{PV_B} \left(\sum_{i=1}^3 i PV_i \right) = \frac{1}{\$807.85} \left(1 \cdot \frac{\$40}{1.12} + 2 \cdot \frac{\$40}{1.12^2} + 3 \cdot \frac{\$1040}{1.12^3} \right) \approx 2.872 \\ D_C &= \frac{1}{PV_C} \left(\sum_{i=1}^3 i PV_i \right) = \frac{1}{\$711.78} \left(3 \cdot \frac{\$1000}{1.12^3} \right) = 3 \end{aligned}$$

3)

a) Computing the price directly:

$$\begin{aligned} PV'_A &= \frac{\$160}{1.10} + \frac{\$160}{1.10^2} + \frac{\$1160}{1.10^3} && \approx \$1149.21 \\ PV'_B &= \frac{\$40}{1.10} + \frac{\$40}{1.10^2} + \frac{\$1040}{1.10^3} && \approx \$850.78 \\ PV'_C &= \frac{\$1000}{1.10^3} && \approx \$751.31 \end{aligned}$$

b) Using the duration approximation,

$$PV'_A = PV_A(1 - D_A^* \Delta R) \approx \$1096.07 \left(1 + \frac{2.623}{1.12} \cdot 0.02 \right) \approx \$1147.41$$

$$PV'_B = PV_B(1 - D_B^* \Delta R) \approx \$807.85 \left(1 + \frac{2.872}{1.12} \cdot 0.02 \right) \approx \$849.28$$

$$PV'_C = PV_C(1 - D_C^* \Delta R) \approx \$711.78 \left(1 + \frac{3}{1.12} \cdot 0.02 \right) \approx \$749.91$$

c) First, let's compute the convexity of the bonds:

$$V_{m_A} = \frac{1}{PV_A \cdot (1+y)^2} \sum_{i=1}^i (i+1) \cdot PV_i$$

$$\approx \frac{1}{\$1096.07(1.12)^2} \left(1 \cdot 1 \cdot 2 \cdot \frac{\$160}{1.12} + 2 \cdot 3 \cdot \frac{\$160}{1.12^2} + 3 \cdot 4 \cdot \frac{\$1160}{1.12^3} \right) \approx 7.971$$

$$V_{m_B} = \frac{1}{PV_B \cdot (1+y)^2} \sum_{i=1}^i (i+1) \cdot PV_i$$

$$\approx \frac{1}{\$807.85(1.12)^2} \left(1 \cdot 1 \cdot 2 \cdot \frac{\$40}{1.12} + 2 \cdot 3 \cdot \frac{\$40}{1.12^2} + 3 \cdot 4 \cdot \frac{\$1040}{1.12^3} \right) \approx 9.025$$

$$V_{m_C} = \frac{1}{PV_C \cdot (1+y)^2} \sum_{i=1}^i (i+1) \cdot PV_i$$

$$\approx \frac{1}{\$711.78(1.12)^2} \left(3 \cdot 4 \cdot \frac{\$1000}{1.12^3} \right) \approx 9.566$$

Now, using the duration + convexity approximation,

$$PV'_A = PV_A \left(1 - D_A^* \Delta R + V_{m_A} \frac{(\Delta R)^2}{2} \right) \approx \$1096.07 \left(1 + \frac{2.623}{1.12} \cdot 0.02 + 7.971 \cdot \frac{0.02^2}{2} \right) \approx \$1149.16$$

$$PV'_B = PV_B \left(1 - D_B^* \Delta R + V_{m_B} \frac{(\Delta R)^2}{2} \right) \approx \$807.85 \left(1 + \frac{2.872}{1.12} \cdot 0.02 + 9.025 \cdot \frac{0.02^2}{2} \right) \approx \$850.74$$

$$PV'_C = PV_C \left(1 - D_C^* \Delta R + V_{m_C} \frac{(\Delta R)^2}{2} \right) \approx \$711.78 \left(1 + \frac{3}{1.12} \cdot 0.02 \right) \approx \$751.27$$

The approximation with just duration is fairly good — giving an error of approximately 0.3% — because the change in interest rate is small. However, the approximation with duration and convexity is better, with an error below 0.01% for all three bonds.

d) The durations are now

$$D'_A = \frac{1}{PV'_A} \left(\sum_{i=1}^3 i PV_i \right) = \frac{1}{\$1149.21} \left(1 \cdot \frac{\$160}{1.10} + 2 \cdot \frac{\$160}{1.10^2} + 3 \cdot \frac{\$1160}{1.10^3} \right) \approx 2.632$$

$$D'_B = \frac{1}{PV'_B} \left(\sum_{i=1}^3 i PV_i \right) = \frac{1}{\$850.78} \left(1 \cdot \frac{\$40}{1.10} + 2 \cdot \frac{\$40}{1.10^2} + 3 \cdot \frac{\$1040}{1.10^3} \right) \approx 2.876$$

$$D'_C = \frac{1}{PV'_C} \left(\sum_{i=1}^3 i PV_i \right) = \frac{1}{\$751.31} \left(3 \cdot \frac{\$1000}{1.10^3} \right) = 3$$

Problem 3:

1) The implied forward rate is

$$\frac{1.0550^2}{1.0525} - 1 \approx 5.75\%$$

which is higher than the bank's rate of 5.5%.

2) We'd like to sell short 10,000 1-year STRIPS (face value \$10 million), which we'll pay off with the expected \$10 million account in one year. This will provide immediate income of $\frac{\$10,000,000}{1.0525} \approx \$9,501,187$. We'll use those proceeds to purchase 2-year STRIPS (9,501.187 of them, assuming we can trade fractional amounts).

In two years, the 2-year STRIPS will return $9,501,187 \cdot (1.055)^2 \approx 10,575,059$ — which, sure enough, is the 5.75% return we wanted on the \$10,000,000 receivable.

Problem 4:

The duration of a set of bonds is the weighted average of their durations, and the duration of a zero-coupon bond is its maturity, so we want portfolio weights of 70% 6-year bonds and 30% 16-year bonds to give a portfolio duration of 9 years.

The price of a 6-year zero-coupon bond is $\frac{\$100}{1.075^6} \approx \64.80 and that of a 16-year bond is $\frac{\$100}{1.075^{16}} \approx \31.44 . Valerie should purchase $70\% \cdot \$500,000 = \$350,000$ worth of the 6-year bonds, and $\$150,000$ of the 16-year bonds. That amounts to 5402 6-year bonds, and 4771 16-year bonds.

