



---

Problem Set 3

Problem 1:

$$P_A = \sum_{t=1}^{\infty} \frac{\$2.20}{1.095^t} = \frac{\$2.20}{0.095} \approx \$23.16$$
$$P_B = \sum_{t=1}^{\infty} \frac{\$1.40 \cdot (1.03)^{t-1}}{1.095^t} = \frac{\$1.40}{0.095 - 0.03} \approx \$21.54$$
$$P_C = \frac{\$1.00}{1.095} + \frac{\$1.50}{1.095^2} + \frac{\$2.00}{1.095^3} + \frac{\$2.50}{1.095^4} + \sum_{t=5}^{\infty} \frac{\$2.50}{1.095^t}$$
$$\approx 0.913 + 1.251 + 1.523 + 1.739 + \frac{1}{1.095^4} \frac{\$2.50}{0.095} \approx \$23.73$$

Problem 2:

1) The firm has a payout ratio of 0.4 and a plowback ratio of 0.6. If it maintains its ROE forever, it will grow at a rate of

$$g = b \cdot ROE = 0.6 \cdot 0.1 = 6\%$$

. Therefore, its stock is valued at

$$P_0 = \frac{D_1}{r - g} = \frac{1 + g}{r - g} D_0 = \frac{1.06}{0.08 - 0.06} \$0.40 = \$21.20$$

Its forward PE ratio is

$$P_0 / EPS_1 = \frac{P_0}{(1 + g)EPS_0} = \frac{\$21.20}{1.06 \cdot \$1} = 20$$

which is higher than  $1/r$  because the firm's PVGO is positive.

2) The firm maintains a ROE of 10% for ten years — we'll interpret that to mean that the ROE falls to 6% and the payout ratio increases to 1 after year 10. Therefore, its book value, earnings, and dividend will increase as above for years 1 - 10.

At the present time, the book value of the firm is \$10 per share (earnings / ROE). The book value will grow by 6% for the first ten years, so after year ten the book value will be  $\$10 \cdot 1.06^{10} \approx \$17.91$  per share. Thereafter, it will not grow, because a payout ratio of 1 implies no further retained earnings. Because the ROE drops to 6%, earnings (and thus dividends) will fall to  $D_{11} = 0.06 \cdot \$17.91 \approx \$1.075$  per share.

$$\begin{aligned}
P_0 &= \sum_{i=1}^{10} \frac{D_i}{1.08^i} + \sum_{i=11}^{\infty} \frac{D_{11}}{1.08^i} \\
&= \frac{(1+g)D_0}{r-g} \left[ 1 - \left( \frac{1+g}{1+r} \right)^{10} \right] + \frac{1}{1.08^{10}} \frac{D_{11}}{0.08} \\
&= \frac{1.06 \cdot \$0.40}{0.02} \left[ 1 - \left( \frac{1.06}{1.08} \right)^{10} \right] + \frac{1}{1.08^{10}} \frac{\$1.075}{0.08} \\
&\approx \$3.614 + \$6.221 \qquad \qquad \qquad \approx \$9.84
\end{aligned}$$

3) Without retention (i.e. a payout ratio of 1), the stock will pay a dividend of \$1 per share for years 1-10. Thereafter, with the reduced ROE, it will pay out a dividend of \$0.6 per share. Thus, its price would be

$$\begin{aligned}
P &= \sum_{i=1}^{10} \frac{\$1}{1.08^i} + \sum_{i=11}^{\infty} \frac{\$0.6}{1.08^i} \\
&= \frac{\$1}{0.08} \left[ 1 - \left( \frac{1}{1.08} \right)^{10} \right] + \frac{1}{1.08^{10}} \frac{\$0.60}{0.08} \\
&\approx \$6.710 + \$3.474 \approx \$9.72
\end{aligned}$$

The PVGO is the difference in price caused by retained earnings, i.e.  $\$9.84 - \$9.72 = \$0.12$ .

**Problem 3:**

We'll start by computing the starting book value, EPS, dividend, and amount reinvested for each of the first five years:

Year	1	2	3	4	5
Book value/share	\$100.00	\$116.00	\$132.70	\$146.64	\$156.90
ROE	20%	18%	15%	10%	8%
EPS	\$20.00	\$20.88	\$19.91	\$14.66	\$12.55
Payout Ratio	20%	20%	30%	30%	60%
Dividend	\$4.00	\$4.18	\$5.97	\$4.40	\$7.53
Reinvested	\$16.00	\$16.70	\$13.93	\$10.26	\$5.02

1)

$$\begin{aligned}
P_0 &= \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \frac{D_4}{(1+r)^4} + \frac{1}{(1+r)^4} \cdot \frac{D_5}{0.07 - 0.08 \cdot 0.6} \\
&\approx \frac{\$4.00}{1.07} + \frac{\$4.18}{1.07^2} + \frac{\$5.97}{1.07^3} + \frac{\$4.40}{1.07^4} + \frac{1}{1.07^4} \cdot \frac{\$7.53}{0.07 - 0.08 \cdot 0.4} \\
&\approx 3.74 + 3.65 + 4.87 + 3.36 + 151.20 \qquad \qquad \qquad \approx \$166.82
\end{aligned}$$

2) If the firm pays out its entire earnings each year, the price would be

$$\frac{\$20}{1.07} + \frac{\$18}{1.07^2} + \frac{\$15}{1.07^3} + \frac{\$10}{1.07^4} + \frac{1}{1.07^5} \frac{\$8}{0.07} \approx \$135.77$$

so the PVGO is

$$PVGO = \$166.82 - \$135.77 \approx \$31.05$$

3)

$$P/E = \frac{\$156.93}{\$20} = 8.34$$

4) In year 5, the firm has EPS of \$12.55, and a growth rate  $g = 8\% \cdot 40\% = 3.2\%$ . The EPS in year 11, therefore, is

$$EPS_{11} = \$12.55 \cdot 1.08^{(11-5)} \approx 15.16$$

and so the dividends will be

$$D_{11} = 0.6 \cdot EPS_{11} \approx \$9.10$$

The ex-dividend stock price will be

$$P_{10} = \frac{D_{11}}{r - g} = \frac{\$9.10}{0.07 - 0.032} \approx \$239.38$$

The forward P/E ratio is

$$P/E = \frac{P_{10}}{EPS_{11}} \approx \frac{\$239.38}{\$15.16} \approx 15.79$$

The PVGO is

$$PVGO = P_{10} - P_{10}^* = \frac{D_{11}}{r - g} - \frac{EPS_{11}}{r} \approx \$239.38 - \frac{\$15.16}{0.07} \approx \$239.38 - \$216.57 \approx \$22.81$$

#### Problem 4:

Consider the possibilities for a 45% drop in price given the the valuation formula

$$P_0 = \frac{D_1}{r - g}$$

where  $D_1$  is the expected dividend for the next year,  $r$  is the discount rate (including risk premium), and  $g$  is the (constant) dividend growth rate:

One possible explanation for 2008–2009's 45% drop in price is that the expected dividend for the next year and all future years dropped. However, they would have to drop by 45% to completely explain the price drop. Although dividends were lower in 2009 than 2008, they fell by only about 20%. So this clearly isn't the whole story.

A more likely explanation is that the expected rate of dividend growth changed as a result of updated information. Consider the amount of change  $\Delta g$  required to change the price by 45%:

$$\begin{aligned} P(1 - .45) &= \frac{D_1}{r - g - \Delta g} \\ \frac{D_1}{r - g} \cdot 0.55 &= \frac{D_1}{r - g - \Delta g} \\ \Delta g &= -\frac{.45}{.55}(r - g) \end{aligned}$$

We can estimate  $r - g$  from previous data, given that  $r - g = \frac{D_{x+1}}{P_x}$  in year  $x$ . For the previous three years,  $r - g$  was approximately  $2\% \pm 0.1\%$ . So a change of

$$\Delta g = -\frac{.45}{.55}(r - g) = -1.6\%$$

in the dividend growth rate would give a 45% drop in the stock price. A 1.6% drop in the growth rate seems like a fairly reasonable expectation given the events of 2008.

A combination of these two factors — a drop in the next year's expected dividend rate and a slowdown of future dividend growth — could provide a better explanation. For example, if dividends were expected to fall by 20% (as they did), a drop of only 0.6% in the dividend growth rate would explain the 45% decrease in stock prices.