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Finance Theory

2011/10/31

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Problem Set 4

Problem 1:

1) The coffee producer will sell three futures contracts: one for 150,000 lbs coffee to be delivered at the end of this month, and one for each of the following months. The prices of the future contracts are (for 150,000 pounds):

$$\begin{aligned}
F_{0,1} &= S_0\,(1+r-\hat{y})^{1/12} && = 150,000 \cdot \$2.90(1+0.005-0.04)^{1/12} \approx \$433,710\\
F_{0,2} &= S_0\,(1+r-\hat{y})^{2/12} && = 150,000 \cdot \$2.90(1+0.005-0.04)^{2/12} \approx \$432,425\\
F_{0,3} &= S_0\,(1+r-\hat{y})^{3/12} && = 150,000 \cdot \$2.90(1+0.005-0.04)^{3/12} \approx \$431,143
\end{aligned}$$

The producer will receive revenue of \$433,710, \$433,425, and \$431,143 in the next three months respectively.

2) Forwards are individualized contracts with a specific counterparty, while futures are standardized contracts traded through a clearinghouse. This means that a forward contract could be set up with any desired asset or maturity, assuming a counterparty can be found. However, such contracts could be quite illiquid, making it difficult to obtain the necessary forward at a reasonable price; this is unlikely to be the case with futures.

Forwards have a significant counterparty risk: the buyer must account for the possibility that the seller will default, and vice versa. This is unlikely to be the case for futures, as the contracts are guaranteed by the clearinghouse. The clearinghouse's default risk is low because it requires each party to post collateral on a margin account. This can be seen as a disadvantage for futures; with forwards, no money is needed until the settlement date.

Problem 2:

1)

$$\begin{aligned}
&\text{1 month:} && 10741.60 = 10740.71(1+r_1)^{1/12} && \Rightarrow r_1 = 0.10\%\\
&\text{3 months:} && 10749.56 = 10740.71(1+r_1)^{3/12} && \Rightarrow r_3 = 0.33\%\\
&\text{6 months:} && 10780.91 = 10740.71(1+r_1)^{6/12} && \Rightarrow r_6 = 0.75\%\\
&\text{12 months:} && 10839.52 = 10740.71(1+r_1)^{12/12} && \Rightarrow r_{12} = 0.92\%
\end{aligned}$$

2) To obtain today's price for the three months of copper production in Q1 2012, sell 80,000 contracts of each of the Jan-12, Feb-12 and Nov-11 copper futures. Revenue will be

Jan-12	$80,000 \cdot \$10769.74 = \$861,579,200$
Feb-12	$80,000 \cdot \$10780.91 = \$862,472,800$
Mar-12	$80,000 \cdot \$10790.75 = \$863,260,000$

3) If copper prices increase to \$20,000, the Jan-12 futures position will have a loss of $80,000 \cdot (\$20,000 - 10769.74) = \$738,420,800$.

If copper prices fall to \$3000, the Jan-12 futures position will realize a gain of $80,000 \cdot (\$10769.74 - \$3000) = \$621,579,200$.

Problem 3:

1)

$$\begin{aligned}
&\text{1 month:} && 10807.08 = 10740.71(1+0.001-\hat{y}_1)^{1/12} && \Rightarrow \hat{y}_1 \approx -7.57\% \\
&\text{6 months:} && 11186.06 = 10740.71(1+0.001-\hat{y}_1)^{6/12} && \Rightarrow \hat{y}_6 \approx -7.71\% \\
&\text{12 months:} && 11186.06 = 10740.71(1+0.001-\hat{y}_1)^{12/12} && \Rightarrow \hat{y}_{12} \approx -8.80\%
\end{aligned}$$

2) A strategy for speculating on copper is as follows: borrow \$955,256,800 from the market. Use this money to purchase 80,000 tons of copper at the spot price (cost: $80,000 \cdot 10740.71 = \$859,256,800$) and pay the storage fee (cost: $80,000 \cdot \$1,200 = \$96,000,000$). After one year, sell the copper and repay the loan. Using the implied interest rate of 0.92% from above, the cost of doing so is $\$955,256,800 \cdot 1.0093 = \$964,140,688.24$.

By contrast, if we instead purchased 80,000 tons of Aug-12 copper futures, the cost would be $80,000 \cdot \$11,784.73 = \$942,778,400$. Purchasing the futures is cheaper after taking into account the storage fee and interest rate.

Problem 4:

1) Consider first purchasing a single futures contract while holding the bonds. At the end of the year, the value of the zero-coupon bonds will be $\$337,500 \cdot 1.06 = \$357,750$. The current price of the index future will be $(1+0.06-0.03)^1 \cdot \$1350 = \$1390.50$. Therefore:

- if the index finishes at 1,200, the value of the portfolio will be $\$357,750 + 250 \cdot (1200 - 1390.5) = \$310,125$.
- if the index finishes at 1,400, the value of the portfolio will be $\$357,750 + 250 \cdot (1400 - 1390.5) = \$360,125$.

If instead we had purchased 250 units of the index (and no bonds), the value of the portfolio would be the value of the index plus the dividend yield

- if the index finishes at 1,200: $\$250 \cdot 1,200 + \$337,500 \cdot 3\% = \$310,125$
- if the index finishes at 1,400: $\$250 \cdot 1,400 + \$337,500 \cdot 3\% = \$360,125$

The two portfolios behave the same way.

2) We could implement this change by purchasing \$20,000,000 in S&P 500 futures. This requires purchasing $\frac{\$20,000,000}{250 * \$1350} = 59$ contracts.

After one year, if the index finishes at 1400, the portfolio consists of

- cash from maturing bond: $\$50,000,000 \cdot 1.06 = \$53,000,000$
- cash from dividends: $\$50,000,000 \cdot 0.03 = \$1,500,000$

- cash from settlement of S&P index futures: $59 \cdot 250 \cdot (1400 - 1390.50) = \$140,125$.
- S&P index (held): $\$50,000,000 \cdot \frac{\$1400}{\$1350} = \$51,851,852$

The total value of the portfolio is now $106,491,977. 51.3% of the portfolio is in cash, and 48.7% in the S&P 500.