15.401 **Dan Ports**
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2011/11/9

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Problem Set 5

Problem 1:

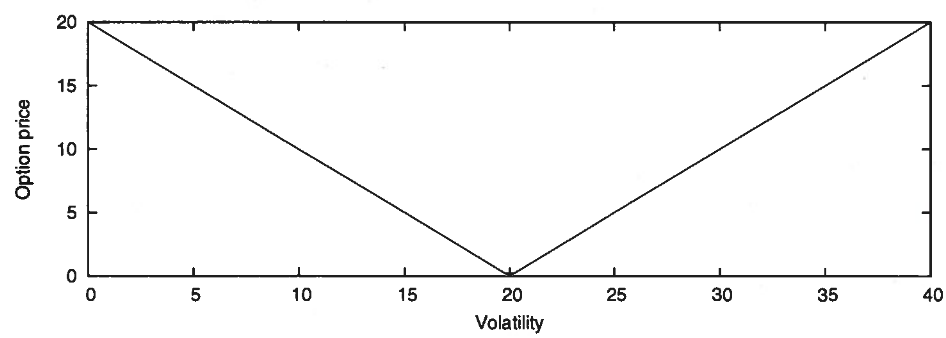
1)

1. Buy 1 call option at 15; sell 1 call option at 25
2. Buy 1 call option at 15; sell 2 call options at 25; buy 1 call option at 30

2) The trader expects the current price to remain steady over the next three months.

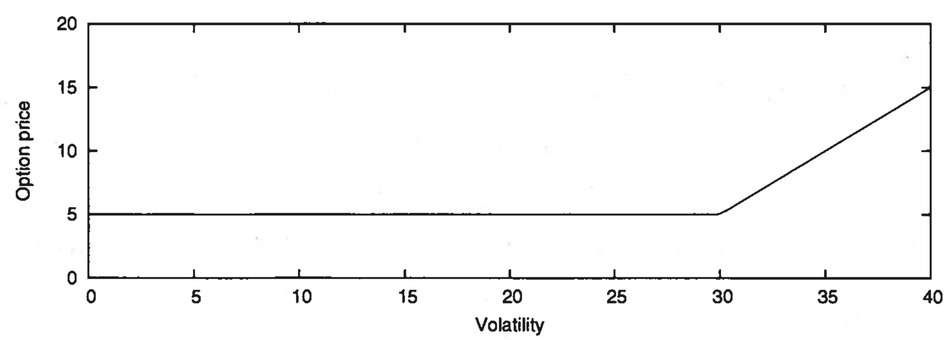
Problem 2:

1)



The trader expects the stock price to change dramatically, but doesn't know the direction.

2)



The trader expects the stock price to increase to a price higher than $30, but wants to hedge the risk that it will not.

Problem 3:

1) Consider first the price of the option at the node ($t = 1$, up). Here,

$$156.25a + 1.05b = 46.25$$
$$93.75a + 1.05b = 0$$

which is solved by $a = 0.740, b \approx -66.071$. The replicating portfolio at this node is long 0.740 shares of stock and short \$66.071 in bonds, which has a cost $0.740 \cdot \$125 - \$66.071 = \$26.43$.

At the node ($t = 1$, down), the value is zero.

Now, at period 0,

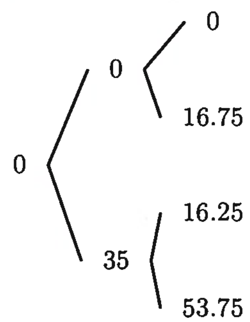
$$125a + 1.05b = 26.43$$
$$75a + 1.05b = 0$$

which is solved by $a = 0.529, b = -37.76$. The replicating portfolio is long 0.529 shares of stock and short \$37.76 in bonds, which has a value $0.529 \cdot \$100 - \$37.76 = \$15.10$.

2) By put-call parity, $S + P = PV(K) + C$, so

$$\begin{aligned} P &= PV(K) + C - S \\ &= \frac{\$110}{1.05^2} + \$15.10 - \$100 \\ &= \$14.87 \end{aligned}$$

3) The exercise value of the put option at each node in the tree is



Consider the ($t = 1$, up) node in the tree. Here,

$$156.25a + 1.05b = 0$$
$$93.75a + 1.05b = 16.25$$

so the replicating portfolio is short 0.2 shares of the stock and long \$38.69 in bonds, giving a value of \$6.19.

At the ($t = 1$, down) node,

$$93.75a + 1.05b = 16.75$$
$$56.25a + 1.05b = 53.75$$

so the replicating portfolio is short 0.987 shares and long \$104.05 in bonds, giving a value of \$30.05. However, this is less than the exercise value of \$35, so we would choose to exercise the option instead, and the value of the option at this node is \$35.

At the $t = 0$ node,

$$125a + 1.05b = 6.19$$
$$75a + 1.05b = 35$$

The replicating portfolio is short 0.576 shares and long \$74.49 in bonds, giving a value of \$16.87.

4) The American put option is worth more than the European one. In the ($t = 1$, down) node we chose to exercise the option early because it gave a higher payoff. The ability to do that makes the American option worth more.

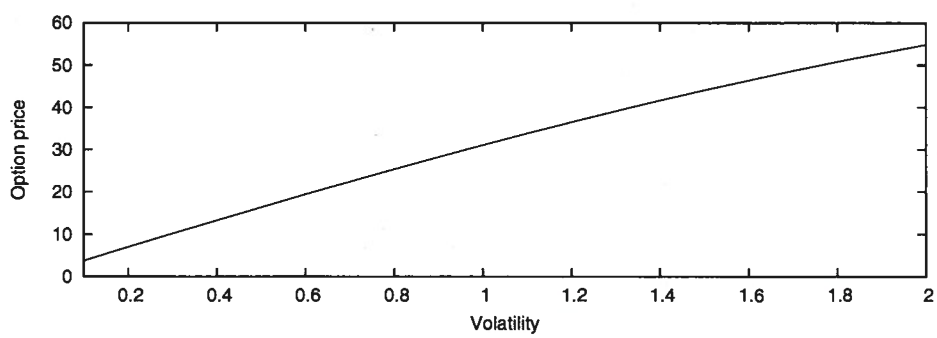
Problem 4:

1)

$$\begin{aligned} x &= \frac{\ln\left(\frac{S}{KR^{-T}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\ &= \frac{\ln\left(\frac{80}{80\cdot 1.02^{-1}}\right)}{\sigma\sqrt{1}} + \frac{1}{2}\sigma\sqrt{1} \\ &= \frac{-0.0198}{\sigma} + \frac{1}{2}\sigma = 0.2104 \\ &= 80\cdot N(x) - 80\cdot 1.02^{-1}N(x - 0.5) \\ &\approx \$16.38 \end{aligned}$$

$$C = SN(x) - KR^{-T}N\left(x - \sigma\sqrt{T}\right)$$

2) The option price is shown below:



As the volatility increases, the value of the option increases. This makes sense, because the option is more valuable as the stock price increases, but has no effect if the stock price drops below its current value. A more volatile stock is more likely to have higher prices, so the option is worth more.