15.401
Finance Theory
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Problem Set 6

Problem 1:

1) False. Systematic risk inherent to all of the assets — for example, due to changing economic conditions — can't be eliminated by diversification.

2) True. If the stock with high standard deviation has a low or negative correlation with the other assets in the portfolio, it can contribute less to portfolio risk.

3) False. In a market with many assets, it might be the case that no individual asset lies on the efficient frontier – meaning that diversification can produce the same returns with lower risk.

4) False. If the return of the dominated asset has a low correlation with the other asset, their combination may have even lower risk.

Problem 2:

1) Yes, because the assets have a low correlation in returns, a portfolio containing some XYZ could have lower risk than one containing only ABC. For example, the 60% ABC / 40% XYZ portfolio discussed below has a lower standard deviation than either asset alone.

2) If the returns are perfectly correlated, there is no longer any reason to invest in XYZ.

3) The expected return of the portfolio is

$$\mathbf{E}[R_p] = \omega_1 R_1 + \omega_2 R_2 = 0.6 \cdot 0.2 + 0.4 \cdot 0.15 = 18\%$$

The standard deviation is given by

$$\begin{aligned}\sigma_p^2 &= \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_1\sigma_2\rho_{1,2} \\ &= 0.6^2 \cdot 0.2^2 + 0.4^2 \cdot 0.25^2 + 2 \cdot 0.2 \cdot 0.4 \cdot 0.2 \cdot 0.25 \cdot 0.2 \\ &= 0.0292 \\ \sigma_p &\approx 17.1\%\end{aligned}$$

4)

$$\begin{aligned}\mathbf{E}[R_p] &= 19.5\% \\ \omega_1 \cdot 0.2 + (1-\omega_1) \cdot 0.15 &= 0.195 \\ \omega_1 &= 0.9\end{aligned}$$

The portfolio should have weights of 90% ABC and 10% XYZ. Its standard deviation is

$$\begin{aligned}\sigma_p^2 &= \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_1\sigma_2\rho_{1,2} \\ &= 0.9^2 \cdot 0.2^2 + 0.1^2 \cdot 0.25^2 + 2 \cdot 0.2 \cdot 0.4 \cdot 0.2 \cdot 0.25 \cdot 0.2 \\ &= 0.034825 \\ \sigma_p &\approx 18.7\%\end{aligned}$$

5)

$$\begin{aligned} 0 &= \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\sigma_1\sigma_2\rho_{1,2} \\ &= \omega_1^2 0.2^2 + (1-\omega_1)^2 0.25^2 - 2\omega_1(1-\omega_1)0.20.25 \\ \omega_1 &\approx 0.555 \end{aligned}$$

The portfolio with 55.5% ABC and 44.5% XYZ would have zero standard deviation.

Problem 3:

1) The Sharpe ratio is maximized at the weights of 77% ABC and 23% XYZ, so this is the tangency portfolio. This portfolio has expected return 18.85% and standard deviation 17.5%.

2)

$$\begin{aligned} 19.5\% &= \omega R_t + (1-\omega)R_f \\ &= \omega 0.1885 + (1-\omega)0.05 \\ \omega &\approx 1.047 \end{aligned}$$

We'd choose a portfolio weight of 1.047 for the tangency portfolio and -0.047 for the risk-free asset (i.e. we're selling the risk-free asset short to leverage our purchase of the tangency portfolio).

Broken down, the portfolio has a weight of 0.806 for ABC, 0.241 for XYZ, and -0.047 for the risk-free asset.

This portfolio has a standard deviation of $17.5\% \cdot 1.047 = 18.32\%$.

3) This portfolio is strictly preferable to the 90% ABC / 10% XYZ portfolio above. It offers the same return (19.5%), with a lower standard deviation.

The difference in the two portfolios is that this one has the same proportions as the tangency portfolio, but uses leverage with the risk-free asset to increase the proportion of risk-free assets and get a higher return. The previous portfolio has to increase the weight of ABC to get a higher return, which means it loses out on some of the benefit of diversification to reduce risk.

Problem 4:

1) The monthly returns are attached. The sample mean and standard deviations are:

	AAPL	**BRK/A**	**BHP**
mean	0.010489364	0.000545504	0.031195384
stddev	0.107696294	0.060310459	0.111027939

The correlation matrix is

	AAPL	**BRK/A**	**BHP**
AAPL	1	0.182381136	0.50051187
BRK/A	0.182381136	1	0.210546788
BHP	0.50051187	0.210546788	1

so the covariance matrix is

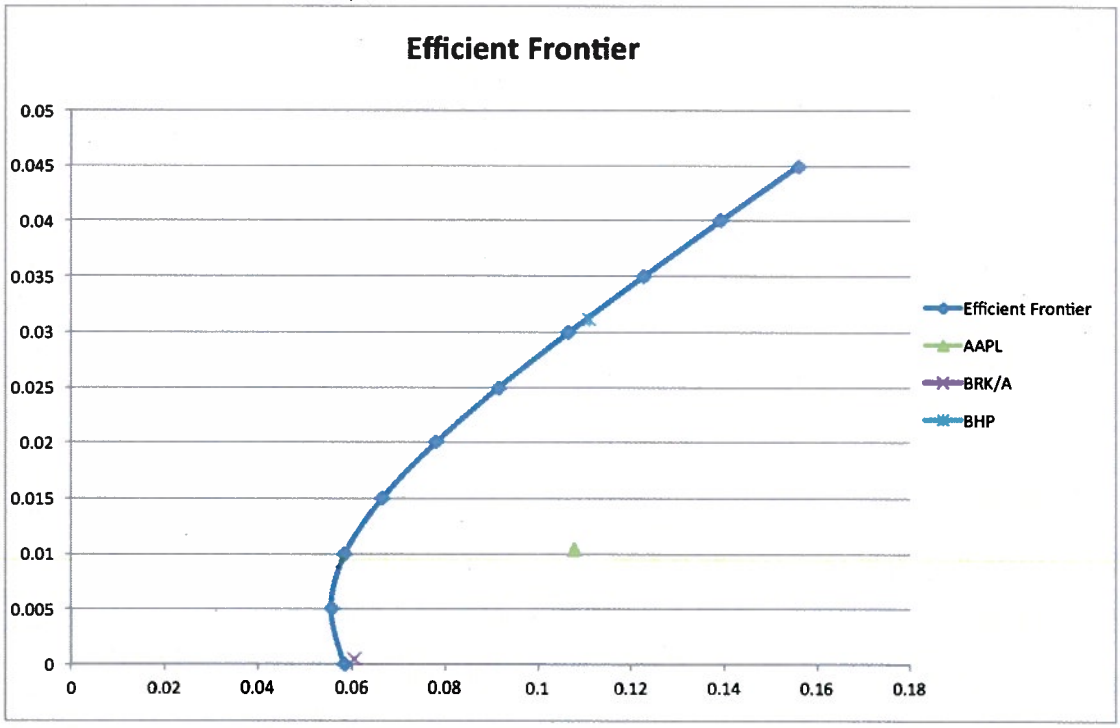
	AAPL	BRK/A	BHP
AAPL	0.011598492	0.001184604	0.005984769
BRK/A	0.001184604	0.003637351	0.001409852
BHP	0.005984769	0.001409852	0.012327203

2)

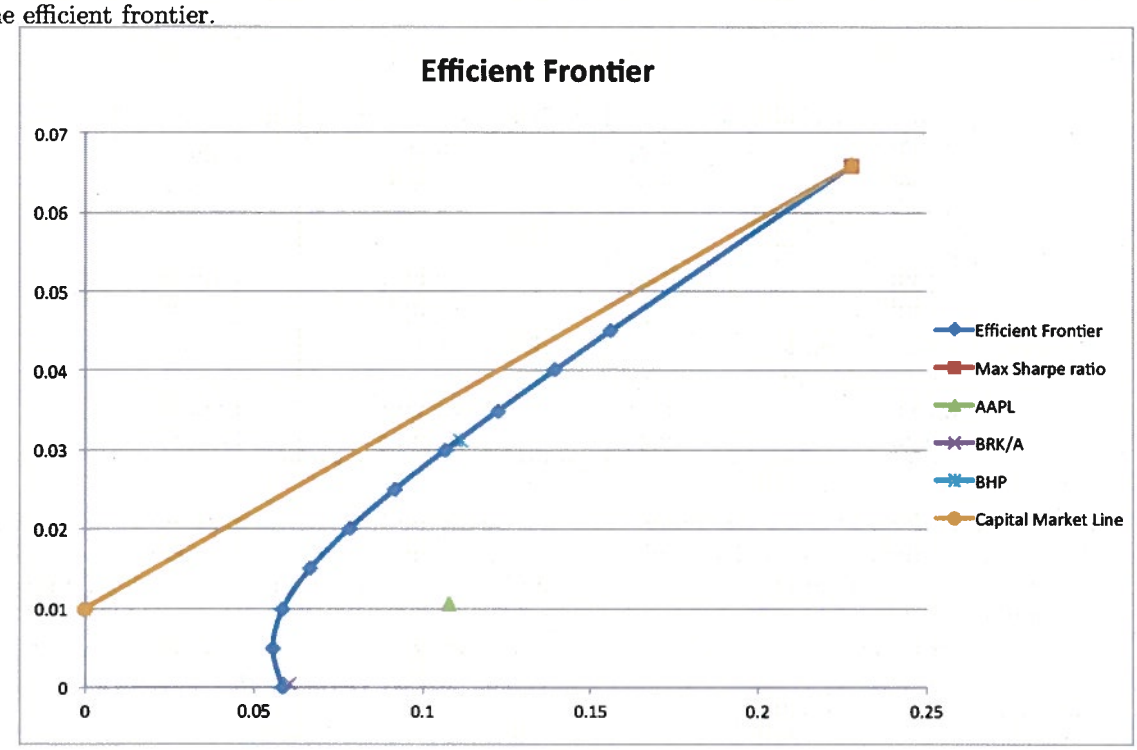
mean	stddev	weight AAPL	weight BRK/A	weight BHP
4.50%	15.59%	-23.50%	-29.16%	152.66%
4.00%	13.91%	-18.85%	-15.99%	134.84%
3.50%	12.27%	-14.20%	-2.82%	117.02%
3.00%	10.68%	-9.55%	10.35%	99.20%
2.50%	9.18%	-4.90%	23.52%	81.38%
2.00%	7.81%	-0.24%	36.69%	63.55%
1.50%	6.66%	4.41%	49.87%	45.73%
1.00%	5.86%	9.06%	63.04%	27.90%
0.50%	5.55%	13.71%	76.20%	10.09%
0.00%	5.83%	18.36%	89.38%	-7.74%

3) The efficient frontier is plotted below. BHP lies nearly on the efficient frontier by itself, with a mean of 3.1% and standard deviation of 11.1%. Compare the 3% mean data point on the efficient frontier, which adds a small short position in AAPL and a long position in BRK/A to reduce diversifiable risk.

Note in particular that the (0.5%, 5.55%) point has both a higher return and lower risk than BRK/A. This is possible because the other two assets, despite having higher standard deviations, have a low correlation with BRK/A.



4) The tangent portfolio uses short positions in AAPL and BRK/A and a long position in BHP, with portfolio weights are −42.87% AAPL, −83.99% BRK/A, and 226.86% BHP. This gives a higher mean (6.58%) and standard deviation (22.8%) than we have seen previously. Not surprisingly, combinations of the risk-free asset and this tangent portfolio dominate the points we saw earlier on the efficient frontier.



Creating efficient frontiers using excel

Suppose we have 3 risky assets whose net returns correspond to the mean vector and variance-covariance matrix given below:

Asset	stddev	Mean	Variance-Covariance Matrix			Weights	Ones
AAPL	10.77%	1.05%	1.16%	0.12%	0.60%	-42.87%	100.00%
BRK/A	6.03%	0.05%	0.12%	0.36%	0.14%	-83.99%	100.00%
BHP	11.10%	3.12%	0.60%	0.14%	1.23%	226.86%	100.00%

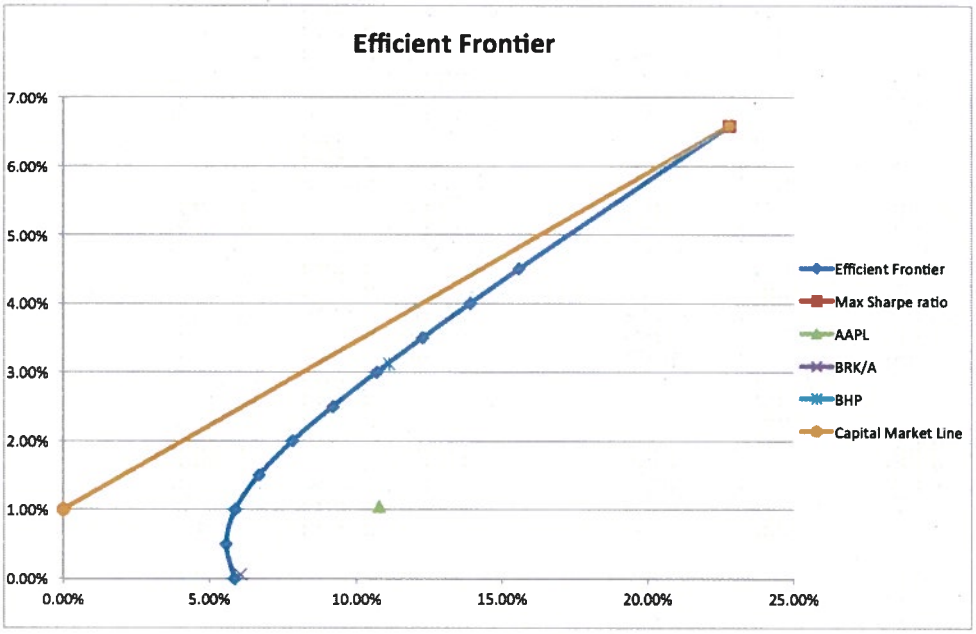
	Mean retur	Variance of	Standard deviation		Constraint
	6.58%	5.20%	22.80%		100.00%

Results

weights		mean	variance	std	
-23.50%	-29.16%	152.66%	4.50%	2.43%	15.59% Note: This line contains points for the minimum variance portfolio.
-18.85%	-15.99%	134.84%	4.00%	1.93%	13.91%
-14.20%	-2.82%	117.02%	3.50%	1.50%	12.27%
-9.55%	10.35%	99.20%	3.00%	1.14%	10.68%
-4.90%	23.52%	81.38%	2.50%	0.84%	9.18%
-0.24%	36.69%	63.55%	2.00%	0.61%	7.81%
4.41%	49.87%	45.73%	1.50%	0.44%	6.66%
9.06%	63.04%	27.90%	1.00%	0.34%	5.86%
13.71%	76.20%	10.09%	0.50%	0.31%	5.55%
18.36%	89.38%	-7.74%	0.00%	0.34%	5.83%
-42.87%	-83.99%	226.86%	6.58%	5.20%	22.80% Note: This line contains points for the maximum Sharpe ratio portfolio.

Graph:

		Risk free	Sharpe Ratio
		0.10%	28.43%
22.80%	6.58%		
15.59%	4.50%		
13.91%	4.00%		
12.27%	3.50%		
10.68%	3.00%		
9.18%	2.50%		
7.81%	2.00%		
6.66%	1.50%		
5.86%	1.00%		
5.55%	0.50%		
5.83%	0.00%		



weights		mean	variance	std
-42.87%	-83.99%	226.86%	6.58%	5.20%	22.80%

Date	AAPL US Equity PRICE - TOT_RET	BRK/A US Equity PRICE - TOT_RETURN	BHP US Equity PRICE - TOT_RETURN_INDEX	AAPL return	BRK/A return	BHP return
2006/10/31	84.2581	105475	64.218			
2006/11/30	95.2528	107100	62.4983	0.130488345	0.015406494	-0.026779096
2006/12/29	88.1655	109990	59.964	-0.074405162	0.026984127	-0.040549903
2007/01/31	89.0903	110050	61.8346	0.010489364	0.000545504	0.031195384
2007/02/28	87.9264	106190	65.3645	-0.013064273	-0.035074966	0.057086162
2007/03/30	96.5518	108990	73.7692	0.098097955	0.026367831	0.128582028
2007/04/30	103.7118	109200	74.363	0.074157085	0.001926782	0.00804943
2007/05/31	125.9413	109490	80.1488	0.214339159	0.002655678	0.077804822
2007/06/29	126.8236	109475	90.9744	0.007005645	-0.000136999	0.135068772
2007/07/31	136.9246	110000	97.1104	0.07964606	0.004795615	0.067447546
2007/08/31	143.908	118390	96.1512	0.051001792	0.076272727	-0.009877418
2007/09/28	159.4855	118510	120.4973	0.108246241	0.001013599	0.253206408
2007/10/31	197.3954	132500	133.7734	0.237701233	0.11804911	0.110177572
2007/11/30	189.3624	140100	116.2508	-0.040694971	0.057358491	-0.130987177
2007/12/31	205.8441	141600	107.3744	0.08703787	0.010706638	-0.076355604
2008/01/31	140.6657	136000	103.5878	-0.316639632	-0.039548023	-0.035265389
2008/02/29	129.9204	140000	113.0773	-0.076388914	0.029411765	0.091608278
2008/03/31	149.1247	133400	101.7511	0.147815893	-0.047142857	-0.10016334
2008/04/30	180.7683	133850	124.6354	0.212195565	0.003373313	0.224904694
2008/05/30	196.1484	134650	130.3217	0.085081842	0.00597684	0.045623475
2008/06/30	174.0031	120750	131.6351	-0.112900742	-0.103230598	0.010078137
2008/07/31	165.1803	114450	115.3642	-0.050704844	-0.052173913	-0.12360609
2008/08/29	176.175	116600	108.9517	0.066561812	0.018785496	-0.055584835
2008/09/30	118.1151	130600	81.6017	-0.32955811	0.120068611	-0.251028667
2008/10/31	111.8072	115490	61.0247	-0.053404687	-0.115696784	-0.252163864
2008/11/28	96.3024	104000	62.814	-0.138674432	-0.099489133	0.029320914
2008/12/31	88.6954	96600	67.3344	-0.078990762	-0.071153846	0.071964849
2009/01/30	93.6628	89502	58.9215	0.056005159	-0.073478261	-0.12494208
2009/02/27	92.8107	78600	58.4506	-0.009097529	-0.121807334	-0.007991989
2009/03/31	109.2404	86700	71.5788	0.17702377	0.103053435	0.22460334
2009/04/30	130.7621	94000	77.2601	0.197012278	0.084198385	0.079371266
2009/05/29	141.1333	91600	90.2598	0.079313501	-0.025531915	0.168258907
2009/06/30	148.0128	90000	87.8364	0.048744697	-0.017467249	-0.026849162
2009/07/31	169.7944	97000	101.0448	0.147160246	0.077777778	0.150375015
2009/08/31	174.8033	100850	99.9856	0.029499795	0.039690722	-0.010482479
2009/09/30	192.6151	101000	107.2558	0.101896246	0.001487357	0.072712471
2009/10/30	195.8886	99000	106.5571	0.016995033	-0.01980198	-0.006514333
2009/11/30	207.7458	100600	122.3506	0.060530322	0.016161616	0.148216308
2009/12/31	218.992	99200	124.4304	0.054134428	-0.013916501	0.016998691
2010/01/29	199.5913	114600	112.7153	-0.088590907	0.155241935	-0.094149822
2010/02/26	212.6404	119800	119.1496	0.065379102	0.045375218	0.057084531
2010/03/31	244.2112	121800	131.8721	0.148470375	0.016694491	0.10677753
2010/04/30	271.3239	115325	119.5091	0.111021526	-0.05316092	-0.093749929
2010/05/31	266.9489	105910	106.4565	-0.016124639	-0.081638847	-0.109218461
2010/06/30	261.3892	120000	101.7773	-0.020826832	0.133037485	-0.043954103
2010/07/30	267.3334	117000	118.5897	0.022740802	-0.025	0.165188112
2010/08/31	252.6287	118675	109.182	-0.055005099	0.014316239	-0.079329824
2010/09/30	294.8721	124500	126.7825	0.167215364	0.049083632	0.161203312
2010/10/29	312.7774	119300	137.1732	0.060722259	-0.041767068	0.081956895
2010/11/30	323.3461	120200	136.8825	0.033789845	0.007544007	-0.002119219
2010/12/31	335.2033	120450	154.3583	0.036670305	0.002079867	0.127670082
2011/01/31	352.6202	122425	147.8963	0.051959214	0.016396845	-0.041863638
2011/02/28	367.0547	131300	157.1491	0.040934978	0.072493363	0.062562755
2011/03/31	362.1679	125300	160.8037	-0.013313547	-0.045696877	0.023255622
2011/04/29	363.854	124750	169.7932	0.004655575	-0.004389465	0.055903564
2011/05/31	361.4638	118775	160.0323	-0.006569118	-0.047895792	-0.05748699
2011/06/30	348.8272	116105	158.7073	-0.034959517	-0.022479478	-0.008279579
2011/07/29	405.7856	111500	153.5417	0.163285432	-0.039662375	-0.032547967
2011/08/31	399.9141	109769	142.8416	-0.014469464	-0.015524664	-0.06968856
2011/09/30	396.2665	106800	113.2737	-0.009120959	-0.027047709	-0.206997821
2011/10/31	420.6461	116950	133.1188	0.061523243	0.095037453	0.175196008

	AAPL	BRK/A	BHP
mean	0.0104894	0.0005455	0.0311954
stddev	0.1076963	0.0603105	0.1110279

Correlation matrix

	AAPL	BRK/A	BHP
AAPL	1	0.1823811	0.5005119
BRK/A	0.1823811	1	0.2105468
BHP	0.5005119	0.2105468	1

Covariance matrix

	AAPL	BRK/A	BHP
AAPL	0.0115985	0.0011846	0.0059848
BRK/A	0.0011846	0.0036374	0.0014099
BHP	0.0059848	0.0014099	0.0123272