15.401

Finance Theory

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Problem Set 7

Problem 1:

1) True. A negative beta means that the investment's risk is not just uncorrelated but *anti*correlated with the market. Holding this investment would reduce the risk of a market portfolio even more than a risk-free asset, so it will have a market price that yields a lower return than the risk-free rate.

2) False. The expected *excess* return on the investment (over a risk-free asset) is twice as high as the expected excess return of the market portfolio.

3) False. Investors demand higher expected rates of return on stocks with a higher beta. It's possible for a stock to have more variable returns, but lower beta because its returns are uncorrelated with the market (as in question 2).

4) False. The CAPM predicts that a security with a beta of zero will offer the same expected return as the risk-free rate.

5) False. Assuming T-bills are risk-free, they have a beta of zero, and the market portfolio has beta 1, so the investor's beta will be $2/3$.

Problem 2:

1)

$$\beta = \frac{\mathbf{Cov}[R_A, R_M]}{\mathbf{Var}[R_M]} = \frac{\phi_{AM}\sigma_A\sigma_M}{\sigma_M^2} = \frac{\phi_{AM}\sigma_A}{\sigma_M} = \frac{0.8 \cdot 0.3}{0.25} = 0.96$$

2)

$$\begin{aligned}\beta &= \frac{\phi_{BM}\sigma_B}{\sigma_M}\\ \phi_{BM} &= \beta\frac{\sigma_M}{\sigma_B}\\ &= 0.8\frac{0.25}{0.4} = 0.5\end{aligned}$$

3)

$$\begin{aligned}\mathbf{E}[R_A] &= \beta_A(\mathbf{E}[R_M] - R_F) + R_F &&= 0.96 \cdot 7\% + 2\% = 7.76\%\\ \mathbf{E}[R_B] &= \beta_B(\mathbf{E}[R_M] - R_F) + R_F &&= 0.8 \cdot 6\% + 2\% = 6.8\%\end{aligned}$$

4) Stock A has a lower standard deviation but a higher beta compared to B. That is, stock B's price is subject to larger fluctuations but they are less correlated with the market price. Thus, B's stock, taken individually, is riskier, but more of its risk is diversifiable.

Problem 3:

1) The expected excess return is 12% - 4% = 8%.

2) We can't say anything about the beta based on the realized return, as the CAPM is about expected return — except inasmuch as realized return can be a predictor for expected return.

3) The stock has an expected excess return (over the risk-free rate) of 10%, so by the CAPM its beta must be $10\%/8\% = 1.25$.

Problem 4:

First, note that the total portfolio value is \$1,388,000, so the portfolio weights of the three stocks are

$$\begin{aligned}
\omega_A &= \frac{10000 \cdot \$25}{\$1,388,000} && \approx 0.1801 \\
\omega_B &= \frac{15000 \cdot \$38}{\$1,388,000} && \approx 0.4107 \\
\omega_C &= \frac{8000 \cdot \$71}{\$1,388,000} && \approx 0.4092
\end{aligned}$$

Now, let's compute the beta of the individual stocks

$$\begin{aligned}
\beta_A &= \frac{\phi_{AM}\sigma_A}{\sigma_M} && = \frac{0.83 \cdot 0.65}{0.21} \approx 2.569 \\
\beta_B &= \frac{\phi_{BM}\sigma_B}{\sigma_M} && = \frac{0.55 \cdot 0.41}{0.21} \approx 1.269 \\
\beta_C &= \frac{\phi_{CM}\sigma_C}{\sigma_M} && = \frac{0.47 \cdot 0.55}{0.21} \approx 1.231
\end{aligned}$$

The beta of the portfolio, therefore, is

$$\begin{aligned}
\beta_P &= \omega_A\beta_A + \omega_B\beta_C + \omega_C\beta_C \\
&\approx 0.1801 \cdot 2.569 + 0.4107 \cdot 1.269 + 0.4092 \cdot 1.231 && \approx 1.488
\end{aligned}$$

So by the CAPM its expected return is

$$\mathbf{E}\left[R_P\right] = \beta_P(11\% - 6\%) + 6\% \approx 13.44\%$$