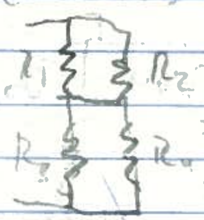
JC $\frac{8}{10}$

Dan Ports
6.002 PS2

(2) A. i. This is a combination of series and parallel resistors.

$$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2+R_3}\right)^{-1} = \boxed{\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}}$$ ✓

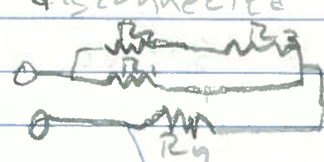
ii. Consider V_S as a short circuit.



By combination of series/parallel circuits,

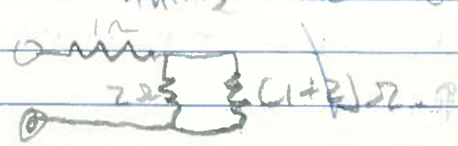
$$R_T = \boxed{\frac{R_1R_2}{R_1+R_2} + \frac{R_3R_4}{R_3+R_4}}$$ ✓

iii. Consider $I_S = 0 \Rightarrow$ open circuit. Then R_S is disconnected and can be ignored



$$\boxed{R_T = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} + R_4}$$ ✓

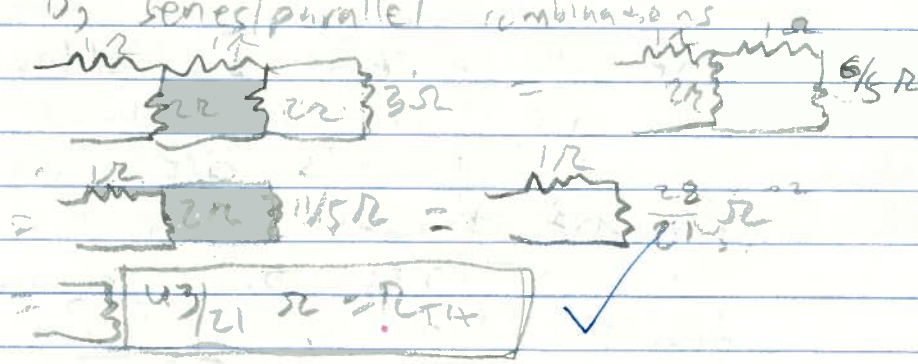
iv. Combining the series resistors on the right gives



Thus $R_T = 1\Omega + \frac{3\Omega \cdot 2\Omega}{3\Omega + 2\Omega}$ ✓

$= 1\Omega + \frac{6}{5}\Omega = \boxed{11/5\,\Omega}$

v. By series/parallel combinations



✓

B. i. $\boxed{R_{N+1} = \left(\frac{2\Omega \cdot R_N}{2\Omega + R_N}\right) + 1\Omega}$ ✓

ii. $\lim_{n\to\infty} R_n = \lim_{n\to\infty} R_{n+1} = R_\infty$

Thus $R_\infty = \frac{2\Omega R_\infty}{2\Omega + R_\infty} + 1\Omega$

$\Rightarrow \cancel{2\Omega R_\infty} + R_\infty^2 = \cancel{2\Omega R_\infty} + 2\Omega^2 + R_\infty \cdot 1\Omega$

$\Rightarrow R_\infty^2 - R_\infty \cdot 1\Omega - 2\Omega^2 = 0$

$\Rightarrow R_\infty = 2\Omega$ or $R_\infty = -1\Omega$ (extraneous)

$\boxed{R_\infty = 2\Omega}$ ✓

The answers from above are approaching R_∞

2. i. $\boxed{R_{Th} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}}$ ✓ from 2.1a(ii),

as the circuits are identical except for the voltage source. By the voltage divider relation,

$\boxed{V_{Th} = V_S \frac{R_2+R_3}{R_1+R_2+R_3}}$ ✓

Short-circuiting the terminals gives

$\boxed{I_N = V_S / R_1}$ ✓

ii. From above $\boxed{R_{Th} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} + R_4}$ ✓

The full current I_S flows through R_5 and returns through R_4 with the terminals open, so the current through R_1 and R_2 must therefore be, by the current divider relation, $I_S \frac{R_3}{R_1+R_2+R_3}$

Thus the voltage drop across R_1 is $I_S \frac{R_1 R_3}{R_1+R_2+R_3}$; and across R_4

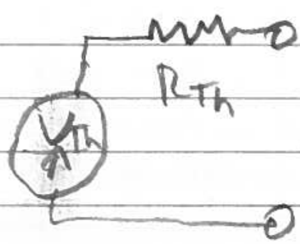
$\Delta V = I_s R_4$, The open-circuit voltage across the terminals is the sum of these

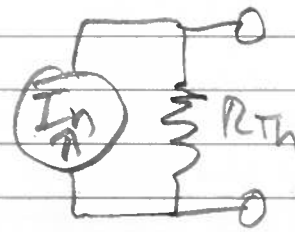
$$\boxed{V_{Th} = V_{oc} = \frac{I_s R_1 R_3}{R_1 + R_2 + R_3} + I_s R_4} \checkmark$$

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{\dfrac{I_s R_1 R_3}{R_1 + R_2 + R_3} + I_s R_4}{\dfrac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3} + R_4} \checkmark$$

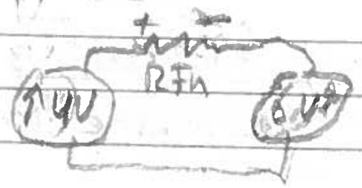
$$\boxed{I_n = \frac{I_s R_1 R_3 + I_s (R_1 + R_2 + R_3) R_4}{R_1 (R_2 + R_3) + (R_1 + R_2 + R_3) R_4}}$$

These two circuits can be replace with either of the following equivalent circuits, with their respective R_{Th}, V_{Th}, and I_n values.



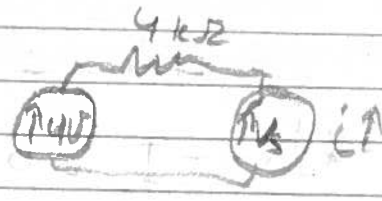


(0) 3a. $V_{Th} = I_1 R = 2mA \cdot 2k\Omega = \boxed{4V}$ ✗

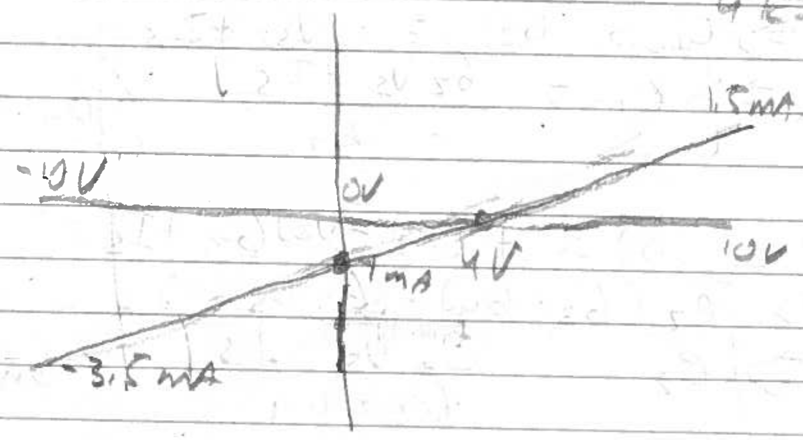


This circuit implies that
$(4-6)V = iR$
$\Rightarrow -2V = -1mA\, R_{Th}$
$\Rightarrow \boxed{R_{Th} = 4k\Omega}$ ✗

b.

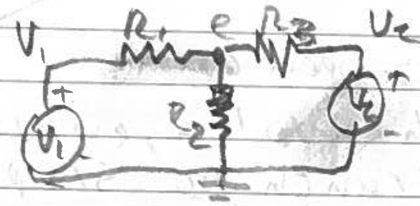


$(V_s - 4V) = i\,(4k\Omega)$
$i = \dfrac{V_s - 4V}{4k\Omega}$



✗

(2) 2.4 a.



$(e - V_1)G_1 + (e - V_2)G_3 + (e - 0)G_2 = 0$
$e(G_1 + G_2 + G_3) = V_2 G_3 + V_1 G_1$
$\boxed{e = \dfrac{V_2 G_3 + V_1 G_1}{G_1 + G_2 + G_3}\ V \quad \text{where } G_n = 1/R_n}$

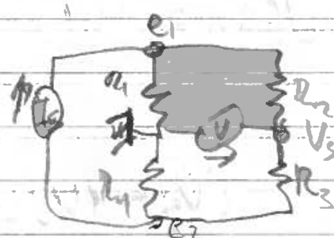
b. Consider V_1 only: V_2 is shorted
$(e' - V_1)G_1 + e' G_2 + e' G_3 = 0$
$\Rightarrow e'(G_1 + G_2 + G_3) = V_1 G_1 \Rightarrow e' = V_1 G_1 / (G_1 + G_2 + G_3)$
Next, with V_2 only, V_1 is shorted
$(e'' - V_2)G_3 + e'' G_2 + e'' G_1$
$\Rightarrow e''(G_1 + G_2 + G_3) = V_2 G_3 \Rightarrow e'' = V_2 G_3 / (G_1 + G_2 + G_3)$

7.4.b (cont)

$$\boxed{e = e' + e'' = \frac{V_1 G_1 + V_2 G_3}{G_1 + G_2 + G_3}} \checkmark \quad (G_n = 1/R_n)$$

This result is the same as part a.

c.



$(e_1 - 0)G_1 + (e_1 - V_s)G_2 - I_s = 0$

$\Rightarrow e_1(G_1 + G_2) = G_2 V_s + I_s$

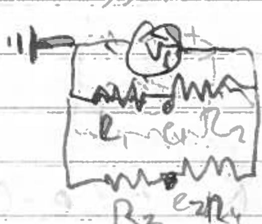
$$\Rightarrow \boxed{e_1 = \frac{G_2 V_s + I_s}{G_1 + G_2}} \checkmark \quad (G_n = 1/R_n)$$

$(e_2 - 0)G_4 + (e_2 - V_s)G_3 + I_s = 0$

$\Rightarrow e_2(G_3 + G_4) = G_3 V_s - I_s$

$$\Rightarrow \boxed{e_2 = \frac{G_3 V_s - I_s}{G_3 + G_4}} \checkmark \quad (G_n = 1/R_n)$$

d. Assume $I_s = 0 \Rightarrow$ open circuit

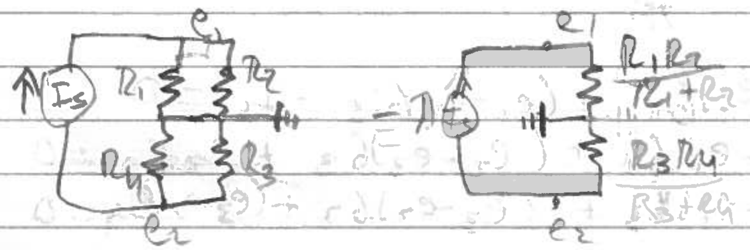


The voltage across each branch is simply V_s, so by the voltage divider relation:

$$e_1' = \frac{V_s R_1}{R_1 + R_2} \qquad e_2' = \frac{V_s R_3}{R_3 + R_4}$$

4d cont'd

Now consider $V_s = 0$ (short circuit)

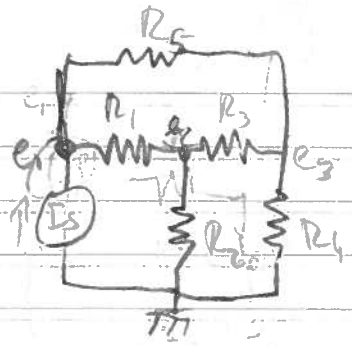


$$e_1'' = \frac{R_1 R_2 I_s}{R_1 + R_2}, \quad e_2'' = -\frac{R_3 R_4 I_s}{R_3 R_4}$$

$$e_1 = e_1' + e_1'' = \frac{V_s R_1 + R_1 R_2 I_s}{R_1 + R_2} \quad \left(= \frac{V_s G_2 + I_s}{G_1 + G_2} \text{ as in part c} \right)$$

$$e_2 = e_2' + e_2'' = \frac{V_s R_4 - I_s R_3 R_4}{R_3 + R_4} \quad \left(= \frac{V_s G_3 - I_s}{G_3 + G_4} \text{ as in part c} \right)$$

(2) S_1



a. $(e_1-e_2)G_1 + (e_1-e_3)G_5 - I_s = 0$
$(e_2-e_1)G_1 + (e_2-e_3)G_3 + e_2 G_2 = 0$
$(e_3-e_1)G_5 + (e_3-e_2)G_3 + (e_3-0)G_4 = 0$

$e_1(G_1+G_5) - e_2 G_1 - e_3 G_5 = I_s$
$-e_1 G_1 + e_2(G_1+G_2+G_3) - e_3 G_3 = 0$
$-e_1 G_5 - e_2 G_3 + e_3(G_3+G_5+G_4) = 0$

$$\begin{bmatrix} G_1+G_5 & -G_1 & -G_5 \\ -G_1 & G_1+G_2+G_3 & -G_3 \\ -G_5 & -G_3 & G_3+G_4+G_5 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \\ 0 \end{bmatrix}$$

b. $$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 5/8 & 3/8 \\ 1/2 & 3/8 & 5/8 \end{bmatrix}\begin{bmatrix} I_s \\ 0 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} I_s/1℧ \\ I_s/2℧ \\ I_s/2℧ \end{bmatrix}}$$

c. The Thevenin-equivalent resistance R_{Th} can be found by applying a test current and finding the voltage required. From above, if a current I_s is applied, then a voltage $I_s/1℧ = I_s \cdot 1\Omega$ is measured across the terminals of the source. Thus $R_{Th} = V/I = I_s \cdot 1\Omega / I_s$ $\boxed{= 1\Omega}$