6/10

Dan Portz
6.002 PS5

1.A. $i_{DS} = K/2 \, (v_{GS} - V_T)^2$

At a few points on the graph, this gives $.5 \text{ mA} = K/2 (3V - V_T)^2$

$2 \text{ mA} = K/2 (4V - V_T)^2$

$4.5 \text{ mA} = K/2 (5V - V_T)^2$

These equations are satisfied by $\boxed{V_T = 2V}$, $K/2 = 1/2 \Rightarrow \boxed{K = 1 \text{ mA}/V^2}$

B. For a, $v_S = 0$, $v_D = V_S - i_{DS} R_1$

$\Rightarrow v_{DS} = V_S - i_{DS} R_1$

For b, $v_S = i_{DS} R_2$, $v_D = V_S$

$\Rightarrow v_{DS} = V_S - i_{DS} R_2$

For c, $v_S = i_{DS} R_2$, $v_D = V_S - i_{DS} R_1$

$\Rightarrow v_{DS} = V_S - i_{DS} R_1 - i_{DS} R_2$

See attached graphs.

C. For a, $v_O = v_{DS}$, $v_I = v_{GS}$

$1/R_1 (V_S - v_{DS}) = K/2 (v_{GS} - V_T)^2$

$\Rightarrow V_S - v_O = \frac{R_1 K}{2} (v_I - V_T)^2$

$\Rightarrow \boxed{v_O = V_S - \frac{R_1 K}{2} (v_I - V_T)^2}$

For b, $v_O = i_{DS} R_2 = V_S - v_{DS}$

$v_I = V_S - v_{DS} + v_{GS} \Rightarrow v_{GS} = v_I - (V_S - v_{DS}) = v_I - v_O$

$1/R_2 (V_S - v_{DS}) = K/2 (v_{GS} - V_T)^2$

$\Rightarrow v_O = R_2 K/2 (v_I - v_O - V_T)^2$

$v_O = R_2 K/2 [(v_I - V_T)^2 - 2 v_O (v_I - V_T) + v_O^2]$

$v_O = R_2 K/2 (v_I - V_T)^2 - R_2 K v_O (v_I - V_T) + \frac{R_2 K}{2} v_O^2$

$\frac{2 v_O}{R_2 K} = (v_I - V_T)^2 - 2(v_I - V_T) v_O + v_O^2$

$v_O^2 - 2(v_I - V_T) v_O - \frac{2}{R_2 K} v_O = -(v_I - V_T)^2$

or $v_O^2 - 2(v_I - V_T + 1/R_2 K) v_O + (v_I - V_T)^2 = 0$

only

$\boxed{v_O = (v_I - V_T - 1/R_2 K) \pm \sqrt{(v_I - V_T + 1/R_2 K)^2 - (v_I - V_T)^2}}$

C. For C, let $R = R_1 + R_2$

$v_O = v_S - iR$

$v_I = v_{GS} + iR_2 \Rightarrow v_{GS} = v_I - iR_2$

$i = \frac{1}{2}k(v_{GS} - v_T)^2$

$i = \frac{k}{2}(v_I - iR_2 - v_T)^2$

$\frac{2}{k} i = (v_I - v_T)^2 - 2(v_I - v_T)iR_2 + i^2R_2^2$

$i^2R_2^2 - 2i\left(R_2(v_I - v_T) + \frac{1}{k}\right) + (v_I - v_T)^2 = 0$

$i = \frac{R_2(v_I - v_T) + \frac{1}{k} \pm \sqrt{(\ldots)^2 - (v_I - v_T)^2 R_2^2}}{R_2^2}$

$i = (v_I - v_T) + \frac{1}{R_2 k} \pm \sqrt{(v_I - v_T + \frac{1}{R_2 k})^2 - (v_I - v_T)^2}$

$v_O = (R_1 + R_2)\left[(v_I - v_T + \frac{1}{R_2 k}) \pm \sqrt{(v_I - v_T + \frac{1}{R_2 k})^2 - (v_I - v_T)^2}\right]$

$v_S - R_1 \mid R_2$

D. For D,

$v_O = (v_I - v_T + \frac{1}{R_2 k}) \pm \sqrt{(v_I - v_T + \frac{1}{R_2 k})^2 - (v_I - v_T)^2}$

For small signal gain, $\frac{1}{2R_2 k}$ can be neglected.

Then the radical term drops out and

$v_O = \frac{1}{?}(v_I - v_T)$

$\frac{dv_O}{dv_I} > 1$

For C,

$v_O = (R_1 + R_2)\left[(v_I - v_T + \frac{1}{R_2 k}) \pm \sqrt{(v_I - v_T + \frac{1}{R_2 k})^2 - (v_I - v_T)^2}\right.$

Similarly, the radical drops out

$v_O = (R_1 + R_2)(v_I - v_T)$

Dan Tortj

For a,

$V_T < V_{GS} < V_{DS} + V_T$

$\Rightarrow V_T < V_I < V_0 + V_T$

For b, $V_T < V_{GS} < V_{DS} + V_T$

$\Rightarrow V_T < V_I - V_0 < V_G - V_0$

For c, $V_T < V_{GS} < V_{DS} + V_T$

$\Rightarrow V_T < V_I - IR_2 < V_0 - IR_2$

$\Rightarrow V_T < V_I - R_2(V_0/R_1+R_2) < V_0 - R_2(V_0/R_1+R_2)$

check this!

(3)

2. A. Assume all MOSFETS are in saturation

Then $2mA = i_x = \frac{k_3}{2}(V_{GS_3} - V_{T_3})^2$

$\Rightarrow 4mA = \frac{1mA}{V^2}(V_{GS_3} - 2V)^2$

$4 = (V_{GS_3} - 2V)^2 \Rightarrow V_{GS_3} - 2V = 2 \Rightarrow V_{GS_3} = 4V$

By the voltage divider relation,

$V_{GS_3} = \frac{20V \cdot R}{8k\Omega + R} = 4V$

$\Rightarrow 20V \cdot R = 8k\Omega \cdot 4V + R \cdot 4V$

$\Rightarrow 16V \cdot R = 32 k\Omega V$

$\Rightarrow \boxed{R = 2k\Omega}$

B. $i_A + i_B = i_C$ The two MOSFETS have the same characteristics and gate voltages. So $i_A = i_B \Rightarrow 2i_A = i_C = 2mA \Rightarrow i_A = 1mA$

$1mA = i_A = \frac{k_1}{2}(V_{GS_1} - V_{T_1})^2 = \frac{2mA}{2V^2}(0 - E_x - 2V)^2$

$\Rightarrow 1V^2 = (-E_x - 2V)^2 \Rightarrow \boxed{E_x = -3V}$

$\boxed{i_A = i_B = 1mA}$

C. $i_A + i_B = 2mA$

$\Rightarrow \frac{2mA}{2V^2}(v_i - e_x - 2V)^2 + \frac{2mA}{2V^2}(-e_x - 2V)^2 = 2mA$

$\Rightarrow (v_i - E_x - e_x - 2V)^2 + (-E_x + e_x - 2V)^2 = 2V^2$

Using the linear approximation,

$(-E_x - 2V)^2 + 2(-E_x - 2V)(v_i - e_x) +$

$+ (-E_x - 2V)^2 + 2(-E_x - 2V)(-e_x) \approx 2V^2$

$\Rightarrow 2(-E_x - 2V)(v_i - e_x) + 2(-E_x - 2V)(-e_x) \approx 0$

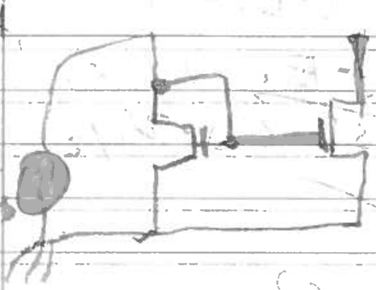
$\Rightarrow (v_i - e_x) + (-e_x) \approx 0 \Rightarrow v_i - 2e_x \approx 0$

$\boxed{\Rightarrow e_x \approx v_i/2}$

$V_O = 10V - i_B \cdot 5k\Omega = 10V - 5k\Omega \cdot \frac{1mA}{1V^2}(e_x - 2V)^2$

Using a linear approximation,

$v_o = \frac{dV_O}{de_x}\Big|_{E_x} \cdot e_x$

3.A 

Consider the first MOSFET. It must satisfy the relation $I = \frac{k}{2}(V_{GS} - V_T)^2$. The current flowing in is I_S, so

$I_S = \frac{k}{2}(V_{GS} - V_T)^2$

We can solve for V_{GS}

$I_S \cdot \frac{2}{k} = (V_{GS} - V_T)^2$

$\Rightarrow V_{GS} = \sqrt{I_S \cdot \frac{2}{k}} + V_T$

The second MOSFET has the same gate voltage and characteristics, so we can solve for its current, i_o.

$i_o = \frac{k}{2}(V_{GS} - V_T)^2 = \frac{k}{2}\left(\sqrt{I_S \cdot \frac{2}{k}} + V_T - V_T\right)^2$

$i_o = \frac{k}{2}\left(\sqrt{I_S \cdot \frac{2}{k}}\right)^2 = \frac{k}{2} I_S \frac{2}{k}$

$\boxed{i_o = I_S}$

The output current is equal to the input current; it mirrors it.

B. There is no change. The same current $I_o = I_S$ flows through the resistor.

Dan Foets

$v_o = \frac{dV_o}{de_x}\Big|_{E_x} \cdot e_x = -5k\Omega \cdot \frac{1mA}{1V^2}(-E_x - 2V)\cdot(-1)\cdot e_x$

$= \frac{10\,k\Omega\cdot mA}{V^2}(-E_x - 2V)\,e_x$

$= 10\,1/V \cdot (3V - 2V)\cdot v_1/2$

$v_o = 5v_1$

$\Rightarrow v_o/v_1 = 5$

D. M_2 will cut off when $V_{GS2} < V_T \Rightarrow (0 - E_x) < 2V$

From above, $(v_1 - e_x - 2V)^2 + (-e_x - 2V)^2 = 2V^2$

$\Rightarrow e_x^2 - 2e_x(v_1 - 2V) + (v_1 - 2V)^2 + e_x^2 + 2e_x 2V + 4V^2 = 2V^2$

$\Rightarrow 2e_x^2 - 2e_x(v_1 - 2V + 2V) + (v_1 - 2V)^2 - 2V^2 = 0$

$\Rightarrow 2e_x^2 - 2e_x v_1 + v_1^2 - 2V\cdot v_1 + 4V^2 - 2V^2 = 0$

$e_x = \frac{2v_1 \pm \sqrt{4v_1^2 - 8v_1^2 - 16V\cdot v_1 + 16V^2}}{4}$

$e_x = \frac{2v_1 \pm \sqrt{-4v_1^2 - 16Vv_1 + 16V^2}}{4}$

$= 1/2\, v_1 \pm \sqrt{-1/4\, v_1^2 - 1V\cdot v_1 + 16V^2}$

Since $-e_x < 2V$ is the cutoff,

$1/2\, v_1 \pm \sqrt{-1/4\, v_1^2 - 1V\cdot v_1 + 16V^2} > -2V$

leads to the cutoff

$V_I = \sqrt{2}\,V$

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