Dan Portz
6.002 PS6

8/10

1.a. $v_A = 10\,V - i_{D_1} \cdot 5k\Omega$

For cutoff, $v_A = 10V$ for $v_{IN} < V_T \Rightarrow V_{IN} < 2V$

In saturation:

$v_A = 10V - 5k\Omega\left(\frac{2mA/V^2}{2} \cdot (v_{IN} - 2V)^2\right)$

$= 10V - 5 \cdot (v_{IN} - 2V)^2/V$

for $v_{IN} > V_T \Rightarrow V_{IN} > 2V$

and $V_{DS} > V_{GS} - V_T \Rightarrow 10V - 5(v_{IN} - 2V)^2/V > V_{IN} - 2V$

$\Rightarrow 10V - 5V_{IN}^2 + 20V_{IN}^2 - 20V > V_{IN} - 2V$

$5V_{IN}^2/V - 20V_{IN} + 10V < 2V - V_{IN}$

$5V_{IN}^2/V - 19V_{IN} + 8V < 0$

$\Rightarrow v_{IN} < 3.3177\,V$

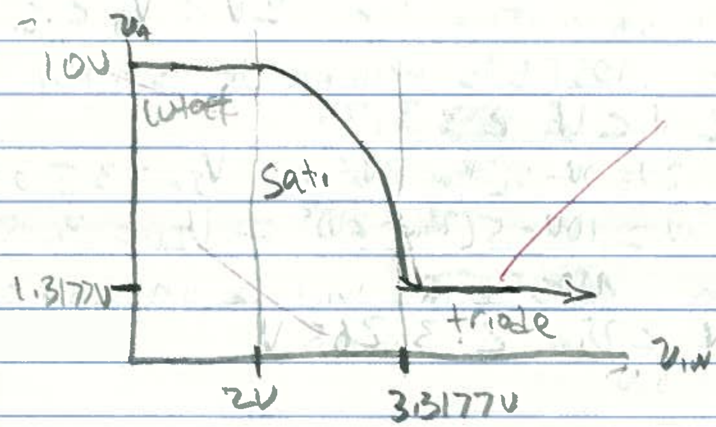
In triode,

$v_A = 10V - 5k\Omega\left(\frac{2mA}{2V^2} \cdot v_A^2\right)$

$\Rightarrow V_A = 10V - 5V_A^2/V \Rightarrow 5V_A^2/V + V_A - 10V = 0$

$\Rightarrow V_A = 1.3177\,V$

for $V_{IN} > 3.3177\,V$

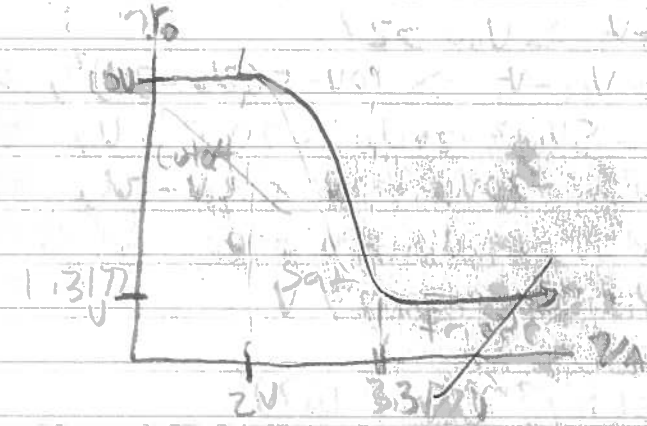


b. By the symmetry of the circuit, the $v_O - v_A$ relation is the same as the $v_A - v_I$ relation.

Thus: $v_O = 10\,V$ for $V_A < 2\,V$ (cutoff)

$v_O = 10\,V - 5(v_A - 2\,V)^2$ for $V_A > 2\,V$ and $V_A < 3.3177\,V$ (sat)

$v_O = 1.3177\,V$ for $V_A > 3.3177\,V$



M_1 will be in saturation for $1.3177\,V < V_A < 10\,V$

M_2 will be in saturation for $2\,V < V_A < 3.3177\,V$

Thus both MOSFETS will be in saturation when $2\,V < V_A < 3.3177\,V$

$v_A = 2\,V \Rightarrow 2\,V = 10\,V - 5(V_{IN} - 2\,V)^2 \Rightarrow V_{IN} = 3.265\,V$

$v_A = 3.3177\,V = 10\,V - 5(V_{IN} - 2\,V)^2 \Rightarrow V_{IN} = 3.156\,V$

Thus both MOSFETS will be in saturation at $3.156\,V < V_{IN} < 3.265\,V$

d. $v_O = 10\,V - 5(V_A - 2\,V)^2\ {}^1/_V$

$= 10\,V - 5((10\,V - 5(v_I - 2\,V)^2) - 2\,V)^2$

$= -125\,v_I^4/V^3 + 1000\,v_I^3/V^2 - 2600\,v_I^2/V$

$+ 2400\,v_I - 710$

e. Let $v_{ID} = 3.2V + v_i$, $v_O = V_O + v_o$

Then $v_o = \frac{dV_O}{dV_I}\Big|_{V_I} \cdot v_i$

$v_o = (-500 V_I^3 + 3000 V_I^2 - 5200 V_I + 2400 V) v_i$

$= (-16384 + 30720 - 16640 + 2400) v_i$

$= 96 v_i$

$\boxed{\frac{v_o}{v_i} = 96}$

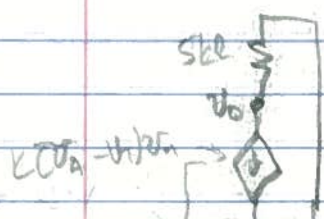
f.

$v_a = k(V_{GS} - V_T) v_{gs} \cdot 5k\Omega$

$= 2mA/V^2 (3.2V - 2V) v_i \cdot 5k\Omega$

$= 10 V/V^2 (1.2V) v_i$

$= 12 v_i$



$v_o = k(V_A - V_T) v_a \cdot 5k\Omega$

$= 2mA/V^2 (2.8V - 2V) v_a \cdot 5k\Omega$

$= 10 V/V^2 (.8V) \cdot (12 v_i)$

$= 96 v_i$

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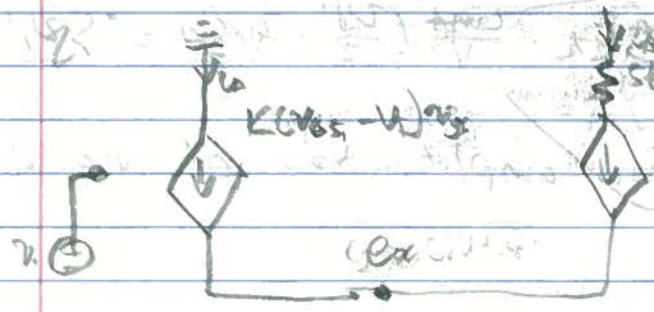
$\boxed{\frac{v_o}{v_i} = 96}$, same gain as above

g.



The bias voltage at the common source of M_1 and M_2 is -3 V $= E_x$ $e_X = E_x + e_x$

$i_a = \frac{2mA}{V^2}(3V - 2V)(v_i - e_x)$

$i_b = \frac{2mA}{V^2}(3V - 2V)(0 - e_x)$

$i_a + i_b = 0 \Rightarrow v_1 - e_x - e_x = 0 \Rightarrow e_x = \frac{1}{2} v_1$

$i_b = \frac{2mA}{V^2}(3V - 2V)(0 - e_x) = 2mA/V \left(-\frac{1}{2} v_1\right) =$

$v_0 = -5k\Omega \cdot i_b = -5k\Omega \cdot -\frac{1}{2} v_1 \cdot \frac{2mA}{V} = 5 v_1$

$\Rightarrow \boxed{\frac{v_0}{v_1} = 5}$

c. $i_a = 2mA/V^2 \, (1V)(-e_x)$

$i_b = 2mA/V^2 \, (1V)(v_1' - e_x)$

$i_a + i_b = 0 \Rightarrow v_1' - e_x - e_x = 0 \Rightarrow v_1' - 2e_x \Rightarrow e_x = v_1'/2$

$i_b = 2mA/V^2 \cdot 1V \cdot (\cancel{e_x} - v_1'/2) = \frac{2mA}{V}(v_1'/2)$

$v_0 = -5k\Omega \cdot i_b = -5k\Omega \cdot 2mA/V \cdot v_1'/2 = -5 v_1'$

$\Rightarrow \boxed{\frac{v_0}{v_1'} = -5}$

d. $i_a = 2mA/V^2 \, (1V)(v_1 - e_x)$

$i_b = 2mA/V^2 \, (1V)(v_1' - e_x)$

$i_a + i_b = 0 \Rightarrow v_1 - e_x + v_1' - e_x = 0 \Rightarrow e_x = \frac{v_1 + v_1'}{2}$

$i_b = \frac{2mA}{V^2} \cdot 1V \left(v_1' - \frac{v_1 + v_1'}{2}\right) = \frac{2mA}{V}\left(\frac{v_1'}{2} - \frac{v_1}{2}\right)$

$v_0 = -5k\Omega \cdot i_b = -5k\Omega \cdot \frac{2mA}{V}\left(\frac{v_1'}{2} - v_1/2\right) = -5v_1' - -5v_1$

$\boxed{v_0 = 5(v_1 - v_1')}$

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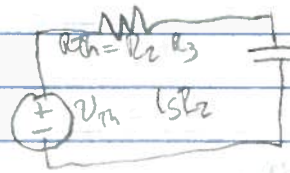
The output is an amplification of the difference of the inputs.

3.a. $C_1 + \frac{C_2(C_3+C_4)}{C_2+C_3+C_4} = C_{eq}$

b. $L_1 + \frac{L_2(L_3+L_4)}{L_2+L_3+L_4} = L_{eq}$

2

4.a. Replace network 1 w/ Thevenin equiv. around the capacitor



At time 0^+, the cap. is a short circuit. $\boxed{v(0^+) = 0}$

At time ∞, the capacitor is open circuit. $\boxed{v(\infty) \Rightarrow V_{Th} = I_s R_2}$

Replace network 2 w/ Norton eq:

At time 0^+ the inductor allows no current flow. $\boxed{i(0^+) = 0}$

At time ∞, the inductor is a short circuit.

$\boxed{i_L(\infty) \Rightarrow I_N = V_1/R_1 - V_2/R_2}$

b. The circuits have time constants $R_{Th}C = (R_1R_2 + R_3)C$ and $L/R_{Th} = L/(R_1R_2/R_1+R_2)$

$\frac{L(R_1+R_2)}{R_1R_2}$

c. $v(t) = I_sR_2 - I_sR_2 e^{-t/(R_1+R_2)C}$

$i(t) = \left(\frac{V_1}{R_1} - \frac{V_2}{R_2}\right) - \left(\frac{V_1}{R_1} - \frac{V_2}{R_2}\right) e^{-t(R_1R_2/R_1+R_2)/L}$

ignored

d. The current $i(0^-)$ at time zero is the current due to v_2 only: $i(0^-) = \frac{V_2}{R_2}$. This is also the current $i(0^+)$ because the inductor resists change. $i(\infty)$ is as before, $\frac{V_1}{R_1}$ ~~$\frac{V_2}{R_2}$~~

Thus the current $i(t)$ is $\frac{V_2}{R_2} + \frac{V_1}{R_1} - \frac{V_1}{R_1} e^{-t/\tau}$

$$\boxed{i(t) = \frac{V_2}{R_2} + \frac{V_1}{R_1}\left(1 - e^{-t\left(\frac{R_1 R_2}{R_1+R_2}\right)/L}\right)}$$

e.

At $t=0^-$, $i_{R_2} = V_2/R_2$

at $t=0^+$, $i_{R_2} = \frac{V_1}{R_1+R_2} + V_2/R_2$

at $t=\infty$, $i_{R_2} = V_2/R_2$

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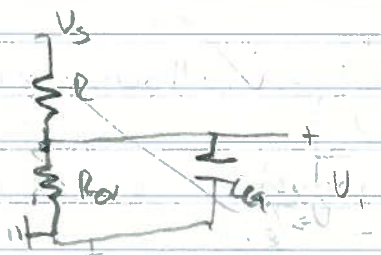
$$\boxed{i_{R_2}(t) = V_2/R_2 + \frac{V_1}{R_1+R_2}\left(e^{-t/\tau}\right)}$$

(where τ is the time constant from above)

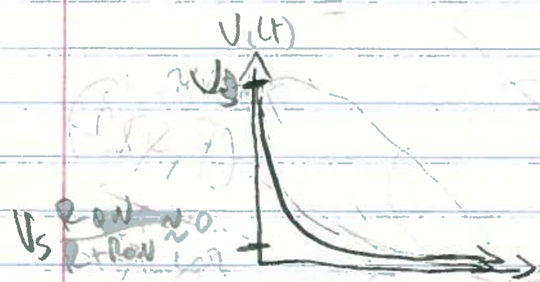
6.5 a.



$v_1(0^+) = V_S$

$v_1(T/2^+) = V_S \frac{R_{ON}}{R+R_{ON}} \approx 0$ because $R >> R_{ON}$

$\tau = (R \| R_{ON}) C_{eq} \approx R_{ON} C_{eq}$ because $R_{ON} << R$.

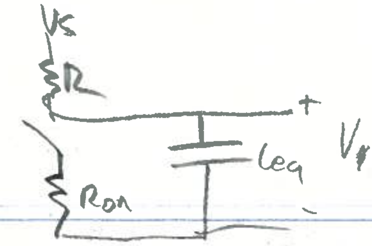


$$\boxed{v_1(t) = V_S\left(e^{-t/R_{ON}C_{eq}}\right)}$$

b. $V_{OL} = V_S\left(e^{-t/R_{ON}C_{eq}}\right) \Rightarrow \frac{V_{OL}}{V_S} = e^{-t/R_{ON}C_{eq}}$

$\Rightarrow \ln\frac{V_{OL}}{V_S} = \frac{-t}{R_{ON}C_{eq}} \Rightarrow \boxed{t = -\ln\left(\frac{V_{OL}}{V_S}\right) R_{ON} C_{eq}}$

c.

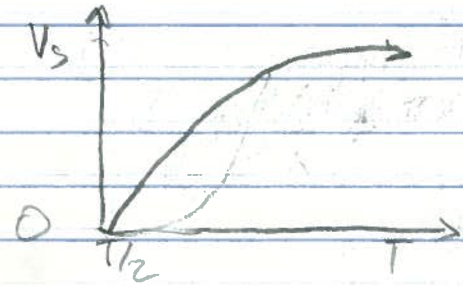


$v_1(T/2) \approx 0$

$v_1(t) = V_s$

$\tau = RC$

$$\boxed{v_1(t) = V_s\left(1 - e^{-t/RC}\right)}$$



$$V_{OH} = V_s\left(1 - e^{-t/RC}\right)$$

$$\frac{V_{OH}}{V_s} = 1 - e^{-t/RC} \Rightarrow e^{-t/RC} = 1 - \frac{V_{OH}}{V_s}$$

(Ron+R)

$$\Rightarrow \frac{-t}{RC} = \ln\left(1 - \frac{V_{OH}}{V_s}\right) \Rightarrow \boxed{t = -RC \ln\left(1 - \frac{V_{OH}}{V_s}\right)}$$

d. Reducing R can allow faster switching time, but it also causes the output low voltage, $V_s\left(\frac{R_{ON}}{R+R_{ON}}\right)$, to increase.

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