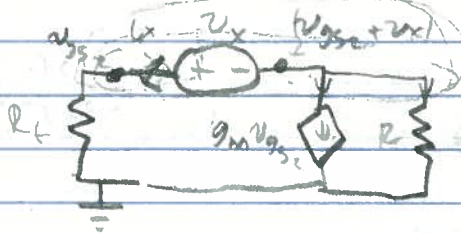


Dan Ports  
6.002 PS7

9/10

1a. To find  $R_{Th}$ , disable all independent sources; apply a test voltage  $v_x$  and find the current through the terminals  $i_x$ .



Applying nodal analysis on the supernode containing  $v_x$ ,  
 $\frac{v_{gs2}}{R_t} + g_m v_{gs2} + \frac{v_{gs2} - v_d}{R} = 0$

$$\Rightarrow \frac{v_{gs2}}{R_t} + g_m v_{gs2} + v_{gs2}/R = v_d/R$$

$$\Rightarrow v_{gs2} \left( \frac{1}{R_t} + g_m + \frac{1}{R} \right) = \frac{v_x}{R}$$

$$v_{gs2} = \frac{v_x}{R \left( \frac{1}{R_t} + g_m + \frac{1}{R} \right)}$$

$$i_x = \frac{v_{gs2}}{R_t} \Rightarrow \frac{v_x}{R_t} = \frac{v_x R_t}{v_{gs2}}$$

$$\Rightarrow \frac{v_x}{i_x} = \frac{v_x R_t + R \left( \frac{1}{R_t} + g_m + \frac{1}{R} \right)}{v_x}$$

$$\frac{v_x}{i_x} = R_t R \left( \frac{1}{R_t} + g_m + \frac{1}{R} \right)$$

$$R_{Th} = \frac{v_x}{i_x} = (R_t + g_m R_t R + R)$$

b.  $T = R_{Th} C_{GD}$  w/  $R_{Th}$  as above

$$T = (R_t + g_m R_t R + R) C_{GD}$$

c.  $v_{out}(0) = v_{out}(0^-) = 0$  because  $v_t$  is 0 and there is no voltage or current flow in the

11. As  $t \rightarrow \infty$ , the capacitor is an open circuit. Then  $V_{out}(\infty) = V_{GS}$ .

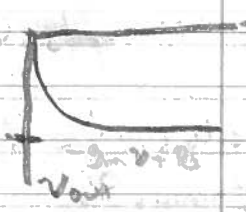
The current flowing through the resistor is provided by the current source.  $I_R = -g_m V_{GS} = -g_m V_t$

Thus  $V_{out} = I_R R = -g_m V_{GS} R$

The time constant is  $\tau = RC$

$$V_{out}(t) = -g_m V_t R (1 - e^{-t/\tau})$$

w/  $\tau$  as above.



12. a.

The capacitor is initially uncharged,  $v(0) = 0$ . Its maximum voltage at large  $t$  is  $V_0/e$ .

$$\text{Thus } v(t) = V_0 (1 - e^{-t/\tau})$$

Here,  $\tau = RC$

b. The capacitor has a final voltage at large  $t$  of  $V_0/e$ . The energy in the capacitor at this time is  $\frac{1}{2} C V_0^2 = \frac{1}{2} C V_0^2$

The voltage source outputs voltage  $V_0$  and current  $V_0/R$  (all time). Integrating  $V I$  all time,  $V_0 \cdot V_0/R \int_0^\infty e^{-t/\tau} dt = \frac{1}{2} C V_0^2$ .

The difference in energy  $\frac{1}{2} C V_0^2$  is dissipated in the resistor.

c. This system has initial current  $I(0) = V_0/R$  because capacitor  $C$  is a short. The resulting current at  $t = \infty$  is  $V_0/eR$ . The time constant of the system is  $\tau = RC_{eq} = R \frac{C}{1+g_m R}$

Thus  $i(t) = V_1/R e^{-t/\tau}$

$v(t) = 1/C_1 \int_0^t i(t) dt = V_1/R C_1 \int_0^t e^{-t/\tau} dt$

$v(t) = \frac{V_1}{R C_1} \tau (1 - e^{-t/\tau}) = \frac{V_1 C_2}{C_1} (1 - e^{-t/\tau})$

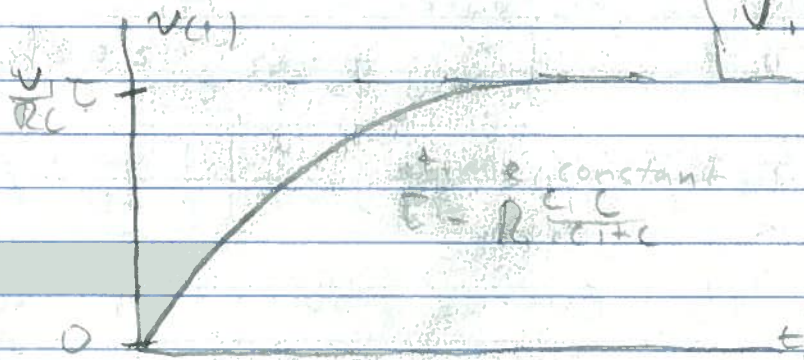
To have the same energy in the capacitor, as  $t \rightarrow \infty$ ,  $v(t)$  must equal  $V_0$ .

Thus  $v(\infty) = V_0$

$\Rightarrow \frac{V_1}{R C_1} \tau (1 - e^{-\infty/\tau}) = V_0$

$\Rightarrow \frac{V_1}{R C_1} \tau = V_0 \Rightarrow V_1 = \frac{V_0 R C_1}{\tau}$

$V_1 = \frac{V_0 (R C_1 + C_1)}{C_1}$



d. The energy dissipated in part C is given by  $\int_0^{\infty} i(t)^2 R dt = \int_0^{\infty} \left( \frac{V_1}{R} e^{-t/\tau} \right)^2 R dt$

$= \int_0^{\infty} R \frac{V_1^2}{R^2} e^{-2t/\tau} dt = \frac{V_1^2}{R} \int_0^{\infty} e^{-2t/\tau} dt = \frac{R V_1^2}{2R^2} R C_2$

$= \frac{1}{2} \frac{V_1^2 C_2}{C_1} = \frac{1}{2} \left( \frac{V_0 (R C_1 + C_1)}{C_1} \right)^2 C_2 = \frac{1}{2} V_0^2 C_2^2$

The energy dissipated in part A is

$\frac{1}{2} V_0^2$  from above. Thus circuit A

dissipates more power by a factor of

$\frac{C_2 (R C_1 + C_1)}{C_1} = \frac{C_1 + C_2}{C_1}$  which is clearly greater than 1

So circuit A is more efficient.

2.  $A + E = 0, \quad i(t) = 0$

At  $t = 0$ , inductor is short and  $i(0) = I_0$

$$\tau = L/R$$

$$i(t) = I_0 (1 - e^{-tR/L})$$

$$v(t) = I_0 R e^{-tR/L}$$

The total energy stored in the inductor is  $\frac{1}{2} L (I_0)^2 = \frac{1}{2} L I_0^2$

The energy dissipated in the resistor is

$$\int_0^{\infty} v i dt = \int_0^{\infty} I_0^2 R e^{-2tR/L} dt$$

$$= \frac{I_0^2 R}{2R/L} (1 - 0) = \frac{1}{2} I_0^2 L$$

3)

The total energy provided by the source is the sum of these.  $L I_0^2$

3a. i From 0 to  $I_0$  the particular solution  $i_p = I_0$ . The homogeneous  $i$  soln is  $i_h = c_1 e^{-tR/L}$

$$(Choose) \quad c \text{ such that } v(0) = 0$$

$$0 = I_0 + c_1 \Rightarrow c_1 = -I_0$$

$$v(t) = I_0 R - I_0 R e^{-tR/L} = I_0 R (1 - e^{-tR/L})$$

$$i(t) = I_0 (1 - e^{-tR/L})$$

$$\text{Thus } v(t) = I_0 R (1 - e^{-tR/L}) e^{-\frac{(L-t)R}{L}}$$

for  $t > T$

a. (ii) In the limit as  $T \rightarrow \infty$ ,

$$\begin{aligned}
 V(t) &= \lim_{T \rightarrow \infty} \frac{1 - e^{-t/T}}{1 + e^{-t/T}} = \frac{(1 - T)/RC}{e^{(1 - T)/RC}} \\
 &= \lim_{T \rightarrow \infty} \frac{1 - T}{1 + e^{-t/T}} = \frac{(1 - T)/RC}{e^{(1 - T)/RC}} \\
 &= \frac{1}{RC} e^{-t/RC} = \boxed{\frac{1}{RC} e^{-t/RC}}
 \end{aligned}$$

b.  $\int_0^t RC \frac{dV}{dt} + \int_0^t V = \int_0^t 1$

$$\Rightarrow RC V(t) + V(0) = 1$$

$$V(0) = \frac{1}{RC}$$

Using the homogeneous solution found in part a, and choosing an appropriate constant,

$$V(t) = \frac{1}{RC} e^{-t/RC}$$

c.  $RC \frac{dV}{dt} = \delta(t)$

$$\int_0^t RC \frac{dV}{dt} = \int_0^t \delta(t) = \frac{1}{RC}$$

$$V(t) = \frac{1}{RC} \int_0^t \delta(t) = \frac{1}{RC}$$

$$V(t) = \frac{1}{RC} e^{-t/RC}$$

d. The step response is the solution to

$$RC \frac{dV}{dt} + V = 1$$

This has a particular solution  $V(t) = 1/RC$ .

To make  $V(0) = V(0)$ , (choose the constant in the homogeneous solution)

approximately

$$v(t) = 1 - e^{-t/RC}$$

The impulse response is the derivative

$$v(t) = 1 - e^{-t/RC} \Rightarrow \frac{dv(t)}{dt} = \frac{1}{RC} e^{-t/RC}$$

Assuming that the charge  $Q$  is applied directly to the capacitor

A.  $v(0^-) = V_0$

$$v(0^+) = v(0^-) + \frac{Q}{C} = V_0 + \frac{Q}{C}$$

After the impulse is delivered, the current source is an open circuit. Thus the voltage across the capacitor is dissipated over the resistor w/ time constant  $RC$

$$v(t) = \left( V_0 + \frac{Q}{C} \right) e^{-t/RC}$$

b.  $v(t) = (V_0 - \frac{Q}{C}) e^{-t/RC}$

Setting this equal to  $v_0 = 0$

$$V_0 - \frac{Q}{C} e^{-t/RC} + \frac{Q}{C} e^{-t/RC} = 0$$

$$V_0(1 - e^{-t/RC}) = \frac{Q}{C} e^{-t/RC}$$

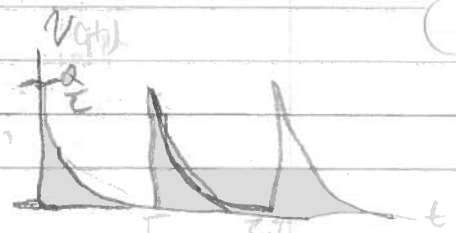
$$V_0 = \frac{Q e^{-t/RC}}{C(1 - e^{-t/RC})}$$

For  $T \gg RC$ ,

$$V_0 \approx \frac{Q \cdot 0}{C} = 0$$

$$v(t) = \frac{Q}{C} e^{-t/RC}$$

$V_0 = 0$



Consider  $\omega = RC \gg T$ . Then

$$V(t) = (V_0 + Q/C) e^{-t/RC} = (V_0 + Q/C) e^0$$

"  $= V_0 + Q/C \Rightarrow V(t)$  is constant.

But we are evaluating  $V(T^-) = V_0$ . Thus

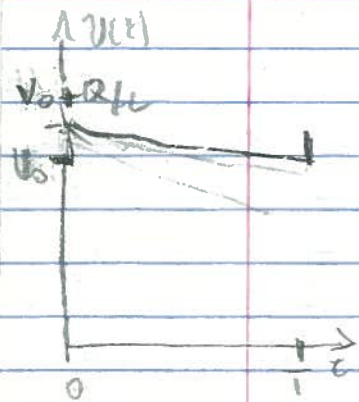
$$V_0 = V_0 + Q/C \Rightarrow Q = 0$$

The only way this can be true is

if  $Q/C = 0 \Rightarrow Q = 0$ , i.e. there

is no impulse. In reality, the situation is not quite so extreme because  $RC$  is not infinitely greater than  $T$ . In this case,  $V(t)$  decays

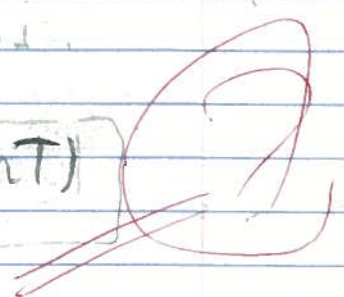
slightly from  $t=0$  to  $t=T$ , and  $V_0 \gg Q/C$



Since the voltage returns to  $V_0$  at every period, the response to this input is also periodic.  $V(t)$  repeats at every period.

$$v = v^*(t) + v(t)$$

$$\text{where } v(t) = \sum_{n=1}^{\infty} v^*(t - nT)$$





The first part of the document discusses the importance of maintaining accurate records. It emphasizes that proper documentation is essential for ensuring the integrity and reliability of the data collected. This section also touches upon the challenges associated with data collection and the need for standardized procedures to overcome these challenges.

In the second part, the focus shifts to the analysis of the collected data. It describes various statistical methods used to interpret the results and identify trends. The author notes that while the data shows promising results, there are still some areas that require further investigation. The conclusion of this section is that the current findings provide a solid foundation for future research in this field.

The final part of the document is a summary of the key findings and a list of references. It reiterates the main points discussed throughout the paper and provides a clear overview of the research. The references cited include several peer-reviewed articles and books that have informed the study. The author expresses their gratitude to the funding agency and the research team for their support and contributions.