JC $\frac{10}{10}$

Dan Portz
6.002 PS 8

2 1.a. At $t=0$, $i(0) = 0$ by continuity
As $t \to \infty$, $i(\infty) \to V/R_1$

$$i(t) = \frac{V_s}{R_1}\left(1-e^{-t/\tau}\right), \quad \tau = \frac{L}{R_1 \| R_2}$$

$$v(t) = -L\frac{di(t)}{dt} = \frac{V_s R_2}{R_1+R_2} e^{-t/\tau}$$

$$\boxed{v_{o_a} = \frac{(1V) R_2}{R_1+R_2} e^{-t/\tau} u(t), \quad \tau = \frac{L(R_1+R_2)}{R_1 R_2}}$$

b. $v_{s_b} = \int_{-\infty}^{t} v_{s_a}(t')\,dt'$

$$\Rightarrow \boxed{v_{o_b} = \int_{-\infty}^{t} v_{o_a}(t')\,dt'}$$

$$= \int_{-\infty}^{t} \frac{1V R_2}{R_1+R_2} e^{-t'/\tau} u(t')\,dt'$$

$$= -\frac{1V R_2}{R_1+R_2}\tau e^{-t'/\tau}\Big|_0^t u(t)$$

$$= -\frac{1V R_2}{R_1+R_2}\tau\left(e^{-t/\tau}-1\right) u(t)$$

$$= \frac{1V R_2}{R_1+R_2}\frac{L(R_1+R_2)}{R_1R_2}\left(1-e^{-t/\tau}\right)u(t)$$

$$\boxed{v_{o_b} = 1V_s \frac{L}{R_1}\left(1-e^{-t/\tau}\right)u(t)}$$

T

c. $v_{s_c}(t) = v_{s_b}(t) - v_{s_b}(t-T)$

by linearity, $v_{o_c}(t) = v_{o_b}(t) - v_{o_b}(t-T)$

$$= 1V_s\frac{L}{R_1}\left[\left(1-e^{-t/\tau}\right)u(t) - \left(1-e^{-(t-T)/\tau}\right)u(t-T)\right]$$

0/0

d. $v_{sd}(t) = v_{sc}(t) + v_{sA}(t)$

$$\boxed{\begin{aligned} &\Rightarrow v_{od}(t) = v_{oc}(t) + v_{oA}(t) \\ &v_{od}(t) = 1V \frac{R_2}{R_1+R_2}\left(e^{-t/\tau}\right)u(t) \\ &\quad + 1V \frac{L}{R_1}\left[\left(1-e^{-t/\tau}\right)u(t) - \left(1-e^{-(t-T)/\tau}\right)u(t-T)\right] \end{aligned}}$$

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e. $v_{se}(t) = v_{sd}(t) - v_{sc}(t-2T) - v_{sa}(t-3T)$

$$\boxed{\begin{aligned} &\Rightarrow v_{oe}(t) = v_{od}(t) - v_{oc}(t-2T) - v_{oa}(t-3\tau) \\ &v_{oe}(t) = 1V \frac{R_2}{R_1+R_2}\left(e^{-t/\tau}\right)u(t) \\ &\quad + 1V \frac{L}{R_1}\left[\left(1-e^{-t/\tau}\right)u(t) - \left(1-e^{-(t-T)/\tau}\right)u(t-T)\right] \\ &\quad - 1V \frac{L}{R_1}\left[\left(1-e^{-(t-2T)/\tau}\right)u(t-2T) - \left(1-e^{-(t-3T)/\tau}\right)u(t-3T)\right] \\ &\quad - 1V \frac{R_2}{R_1+R_2} e^{-(t-3\tau)/\tau} u(t-3T) \end{aligned}}$$

(2) 2a. During the first half-period the capacitor C_{eq} discharges from V_S to $V_{LO} = V_S \frac{R_{ON}}{R+R_{ON}}$. The change in energy stored in the capacitor is

$$\Delta E_C = \tfrac{1}{2} C V_S^2 - \tfrac{1}{2} C \left(V_S \frac{R_{ON}}{R+R_{ON}}\right)^2$$

The energy dissipated is equal to this energy plus the integral of the static power over time $T/2$

$$\boxed{E_{dissipated} = \frac{1}{2} C \left[V_S^2 - \left(V_S \frac{R_{ON}}{R+R_{ON}}\right)^2\right] + \frac{V_S^2}{(R+R_{ON})} \frac{T}{2}}$$

almost

b. During the second period, the voltage source charges the capacitor and dissapates energy through the resistor, so $E_{supplied} = E_{dissipated} + E_{stored}$.

$$E_{supplied} = \int_{T/2}^{T} P_{supplied}\, dt = \int_{T/2} V_S I \, dt$$

$$= \int_{T/2}^{T} V_S \frac{(V_S - V_1)}{R} dt = \int_{T/2}^{T} V_S^2/R - \frac{V_S V_1}{R} dt$$

$$= \frac{V_S^2}{R} \frac{T}{2} - \frac{V_S}{R} \int_{T/2}^{T} V_S - \frac{R}{R_{ON}+R} V_S e^{-(t-T/2)/\tau_2} dt$$

$$= \frac{V_S^2}{R} \frac{T}{2} - \frac{V_S^2 T}{2R} - \frac{V_S^2}{R} \int_0^T \frac{R}{R_{ON}+R} e^{-t/RC} dt$$

$$= \frac{V_S^2}{R_{ON}+R} \cdot RC \left[e^0 - e^{-T/RC}\right.$$

$$= \frac{V_S^2 \cdot RC}{R_{ON}+R}$$

The energy stored in the capacitor is the same as part A, so $E_{dissipated} = E_{supplied} - E_{stored}$

$$\boxed{E_{dis} = \frac{V_S^2 RC}{R_{ON}+R} - \frac{1}{2} C \left[V_S^2 - \left(V_S \frac{R_{ON}}{R+R_{ON}}\right)^2\right]}$$

very good

c. The time-averaged total power is the energies from part a and b added and divided by T

$$\bar{P} = \frac{1}{T}\left[\left(\frac{1}{2}C\left(V_s^2 - \left(\frac{V_s R_{ow}}{R+R_{ow}}\right)^2\right) + \frac{V_s^2}{R+R_{ow}}\frac{T}{2}\right) + \frac{V_s^2 R C_{eq}}{R+R_{ow}} - \frac{1}{2}C\left(V_s^2 - \left(\frac{V_s R_{ow}}{R+R_{ow}}\right)^2\right)\right]$$

$$= \frac{1}{T}\left[\frac{V_s^2}{R+R_{ow}}\frac{T}{2} + \frac{V_s^2 R C_{eq}}{R+R_{ow}}\right]$$

$$= \boxed{\underbrace{\frac{V_s^2}{2(R+R_{ow})}}_{\text{static}} + \underbrace{\frac{V_s^2 R C_{eq}}{(R+R_{ow})T}}_{\text{dynamic}}}$$

d. $$P = \frac{25V^2}{2\cdot 110\ k\Omega} + \frac{25V^2 \cdot 100\ k\Omega \cdot 10^{-2}\ pF}{110\ k\Omega \cdot 10^{-6}\ s}$$

$$= \underbrace{0.114\ mW}_{\text{static}} + \underbrace{0.023\ mW}_{\text{dynamic}} = 0.136\ mW$$

Static dominates

(2)

3 a.

A	B	C	Z
L	L	L	H
L	L	H	L
L	H	L	H
L	H	H	L
H	L	L	H
H	L	H	L
H	H	L	L
H	H	H	L

$Z = \overline{AB + C}$

b

A	B	C	D	Z
L	L	L	L	H
L	L	L	H	H
L	L	H	L	L
L	L	H	H	H
L	H	L	L	H
L	H	L	H	H
L	H	H	L	L
L	H	H	H	H
H	L	L	L	H
H	L	L	H	H
H	L	H	L	L
H	L	H	H	H
H	H	L	L	L
H	H	L	H	L
H	H	H	L	L
H	H	H	H	L

$Z = \overline{AB + C\bar{D}}$

c

A	B	C	D	Z
L	L	L	L	H
L	L	L	H	H
L	L	H	L	L
L	L	H	H	H
L	H	L	L	L
L	H	L	H	H
L	H	H	L	L
L	H	H	H	H
H	L	L	L	H
H	L	L	H	H
H	L	H	L	L
H	L	H	H	L
H	H	L	L	L
H	H	L	H	L
H	H	H	L	L
H	H	H	H	L

$Z = \overline{(A+\bar{D})(B+C)}$

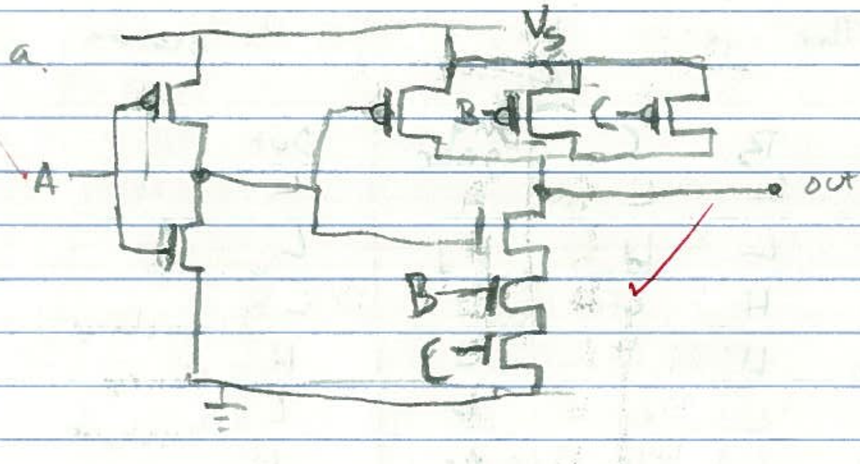
d)

A	B	C	D	Z
L	L	L	L	L
L	L	L	H	L
L	L	H	L	H
L	L	H	H	L
L	H	L	L	H
L	H	L	H	L
L	H	H	L	H
L	H	H	H	L
H	L	L	L	L
H	L	L	H	L
H	L	H	L	H
H	L	H	H	H
H	H	L	L	H
H	H	L	H	H
H	H	H	L	H
H	H	H	H	H

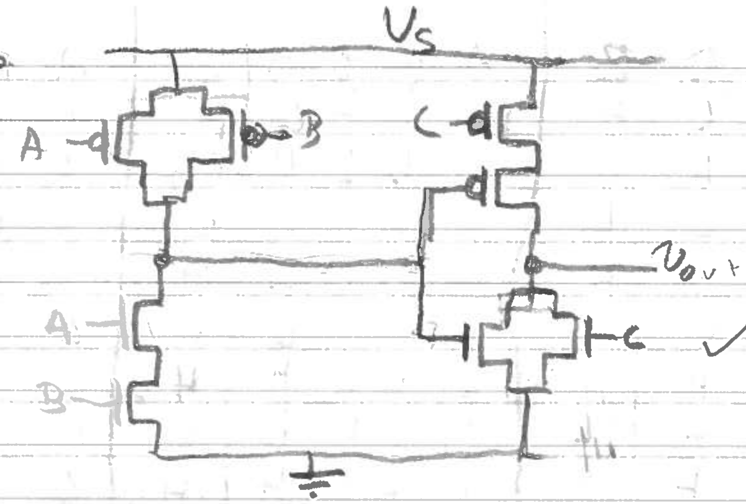
$Z = (A+\bar{D})(B+C)$

2

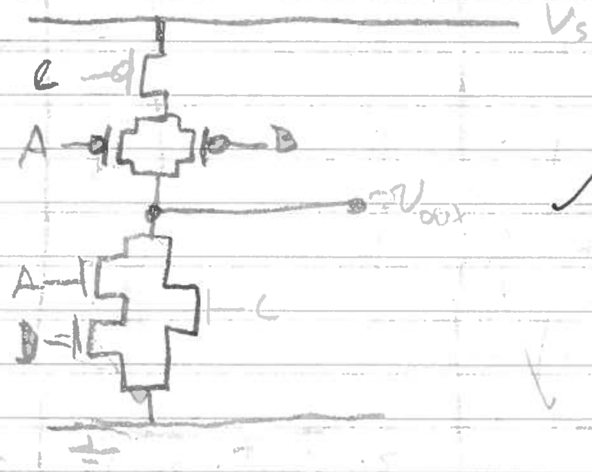
4. a.



b.



c.



② 5. The circuit has the following characteristic

A	B	C	my gate	Out	
L	L	L	H	L	
L	L	H	H	L	
L	H	L	H	L	discharge
L	H	H	L	H	charge
H	L	L	H	L	discharge
H	L	H	L	H	charge
H	H	L	H	L	discharge
H	H	H	L	H	charge

Current is always flowing through one of the R resistors in the steady state because the right branch is the inversion of the left. So there is always a current $V_s / R + R_{on}$ ~~≈~~ V_s / R flowing through one resistor, which means a static power $P = V_s^2 / R$

From the truth table there are 3 L-H transitions in which the capacitor charges and 3 H-L where the capacitor discharges. The energy dissipated while charging to V_s is $\frac{1}{2} C_G V_s^2$, and $\frac{1}{2} C V_s^2$, is lost when discharging. The total dynamic energy change is

$$3 \cdot \frac{1}{2} C_G V_s^2 + 3 \cdot \frac{1}{2} C_G V_s^2 = 3 C_G V_s^2$$

The total power is the static power plus the time-average over 8T of the dynamic ~~energy change~~:

$$\boxed{\bar{P} = V_s^2 / R + \frac{3 C_{GS} V_s^2}{8T}}$$

✓

very good.