MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.002 – Electronic Circuits
Fall 2002

Quiz 3

Name: DAN PORTS Recitation Section: WF 11

Recitation Instructor: VOLDMAN Teaching Assistant: VIKAS

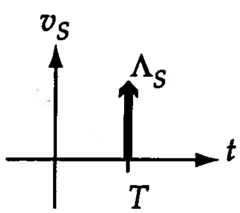
Make sure that your name is on all sheets. Enter your answers directly in the spaces provided on the printed pages. You may use the back of the previous page as a worksheet. Answers must be derived or explained, not just simply written down. The quiz is closed book, but **calculators are allowed.**

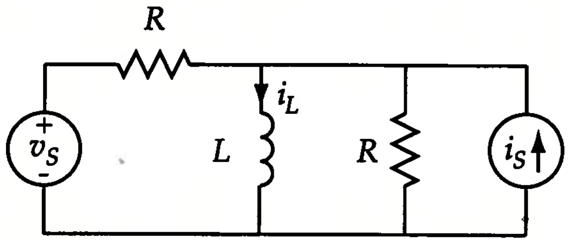
This quiz contains 8 pages including the cover sheet. Make sure that your quiz contains all 8 pages and that you hand in all 8 pages.

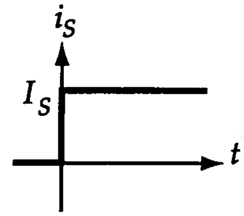
Problem	Points	Grade	Grader
1	30	26	SDU
2	30	30	[illegible]
3	40	38	[illegible]
Total	100	94	

Name: Dan Ports

Problem 1: (*30 points*) This problem examines the transient response of the circuit shown below. In the circuit, $i_L = 0$ at $t = 0^-$.



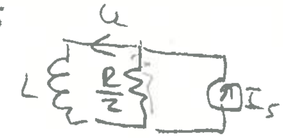




(A) (*10 points*) Determine an expression for i_L due to $i_S = I_S u(t)$; that is, for $\Lambda_S = 0$.

Treat V_S as a short.

Replace the two parallel resistors with their equivalent resistance.



$$i_S = i_R + i_L = \frac{v_L}{R/2} + i_L = \frac{2L}{R}\frac{di_L}{dt} + i_L$$

$$\frac{di_L}{dt} + \frac{R}{2L} i_L = I_S \frac{R}{2L}$$

$$i_{L\,particular} = I_S$$

$$i_{L\,homogeneous} = A e^{-\frac{R}{2L}t}$$

$$i_L(0^-) = i_L(0^+) = 0$$

$$\boxed{i_L = \left(I_S - I_S e^{-(R/2L)t}\right) u(t)}$$

+8

Name: Dan Ports

(B) (*10 points*) Determine an expression for i_L due to $v_S = \Lambda_S \delta(t-T)$; that is, for $I_S = 0$.

Treat I_S as an open. The voltage impulse must be applied to the inductor, since an infinite voltage across a resistor would mean infinity current through the inductor, which is impossible.

$v_S - R(i_R) - v_L = 0$

$\Rightarrow v_S - R(i_L + \frac{v_L}{R}) - v_L = 0 \Rightarrow v_S - i_L R - 2v_L = 0$

$\Rightarrow 2L\frac{di_L}{dt} + Ri_L = v_S \Rightarrow \frac{di_L}{dt} + \frac{R}{2L} i_L = \frac{v_S}{2L}$

So $\boxed{i_L(t) = 0 \quad \text{for } 0 < t < T}$

$\int_{T^-}^{T^+} \frac{di_L}{dt} = \int_{T^-}^{T^+} \frac{\Lambda_S \delta(t-T)}{2L} \Rightarrow i_L(T^+) = \frac{\Lambda_S}{2L}$

After this, $v_S = 0$ so this is a homogeneous response w/ initial condition $i_L(T^+)$ as above:

$$\boxed{i_L(t) = \frac{\Lambda_S}{2L} e^{-\frac{R}{2L}(t-T)}, \quad \text{for } t > T}$$

4/10

(C) (*10 points*) Determine an expression for i_L due to i_S and v_S together.

This is a linear system, so superposition applies:

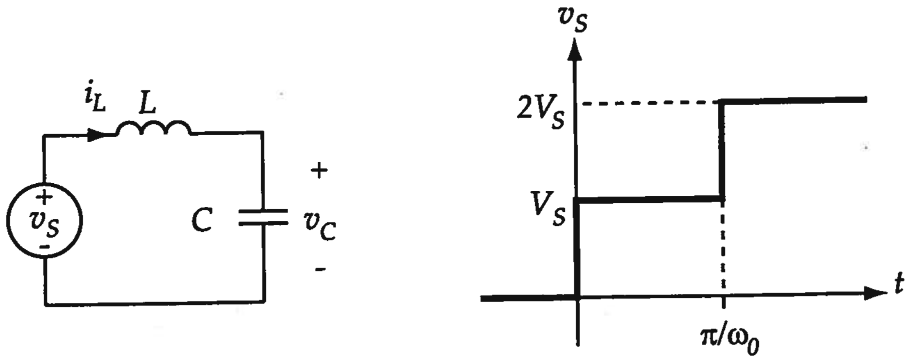
$i_L(t) = i_{LA}(t) + i_{LB}(t)$

$$\boxed{i_L(t) = I_S - I_S e^{-R/2L\, t} + \frac{\Lambda_S}{2L} e^{-\frac{R}{2L}(t-T)} u(t-T)}$$

+8

Name: Dan Portz

Problem 2: (*30 points*) In the circuit below, $\omega_0 \equiv 1/\sqrt{LC}$, $v_C(0^-) = 0$, and $i_L(0^-) = 0$.



(A) (*15 points*) Determine an expression for $v_C(t)$ for $t > 0$. If you determine $v_C(t)$ by inspection, state your reasoning CLEARLY.

$v_s - v_L - v_c = 0 \Rightarrow V_s - L\frac{di_L}{dt} - V_c = 0$

$\Rightarrow V_s - LC\frac{d^2v_c}{dt^2} - V_c = 0 \Rightarrow \frac{d^2v_c}{dt^2} + \frac{v_c}{LC} = \frac{v_s}{LC}$

$v_{C\,homog.} = A\cos(\omega_0 t) + B\sin(\omega_0 t)$

for $t < \pi/\omega_0$, $v_{c\,particular} = V_s$, and $v_c(0^+) = \dot{v}_c(0^+) = 0$

so $\boxed{v_c(t) = V_s - V_s\cos(\omega_0 t) \quad \text{for } 0 < t < \pi/\omega_0}$

$v_c(\pi/\omega_0^+) = v_c(\pi/\omega_0^-) = 2V_s$

$\dot{v}_c(\pi/\omega_0^+) = \dot{v}_c(\pi/\omega_0^-) = 0$

At this point $\frac{d^2v_c}{dt^2} = \frac{v_s(\frac{\pi}{\omega_0}^+) - v_c(\frac{\pi}{\omega_0}^+)}{LC} = \frac{2V_s}{LC} - \frac{2V_s}{LC} = 0.$

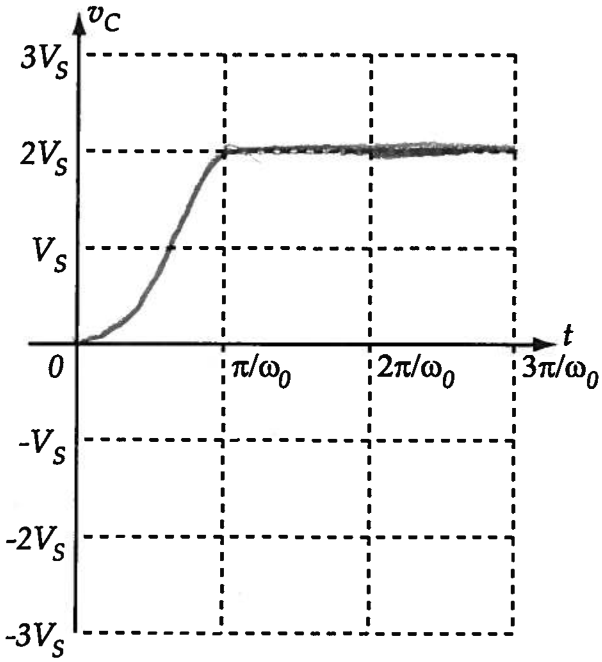
Since the first and second derivatives of v_c are zero and the particular solution is $2V_s$, the coefficients of the homogeneous response are zero and the soln is $\boxed{v_c(t) = 2V_s, \ t > \pi/\omega_0}$

$v_C(t) =$ $V_s - V_s\cos(\omega_0 \cdot t)$ $\quad 0 \le t \le \pi/\omega_0$

$v_C(t) =$ $2V_s$ $\quad t > \pi/\omega_0$

+15

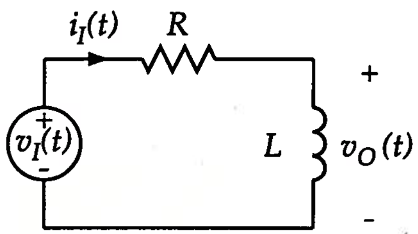
(B) (*15 points*) Sketch and dimension $v_C(t)$ for $t > 0$.

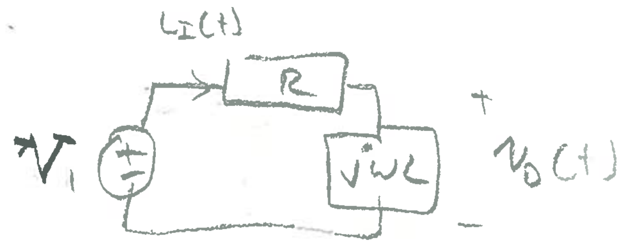


+15

Problem 3: (*40 points*) All circuits below operate in the sinusoidal steady state. For each circuit, determine the input impedance $Z(j\omega)$, the specified transfer function $H(j\omega)$, and the parameters V_o and ϕ which define $v_O(t) \equiv V_o \cos(\omega t + \phi)$, where V_o is a real, positive number.

(A) (*10 points*)





$v_I(t) = V_i \cos(\omega t)$

$Z(j\omega) = R + j\omega L$ by series combination

$$H(j\omega) = \frac{\hat{V}_o}{V_I} = \frac{j\omega L}{Z_L}$$ by voltage divider rel

$$= \frac{j\omega L}{R + j\omega L}$$

Therefore $\hat{V}_o = \frac{j\omega L}{R + j\omega L} \cdot V_I = \frac{\omega L e^{j\pi/2}}{\sqrt{(\omega L)^2 + R^2}\, e^{j\arctan(\frac{\omega L}{R})}} V_I$

$$\Rightarrow V_1(t) = \frac{\omega L V_I}{\sqrt{(\omega L)^2 + R^2}} \cos(\omega t + \varphi)$$

$\varphi = \pi/2 - \arctan(\omega L/R)$

$Z(j\omega) = \frac{V_i}{\hat{I}_i} =$ $R + j\omega L$ ✓

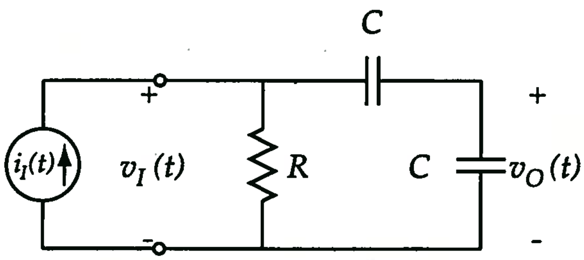
$H(j\omega) = \frac{\hat{V}_o}{V_i} =$ $\frac{j\omega L}{R + j\omega L}$ ✓

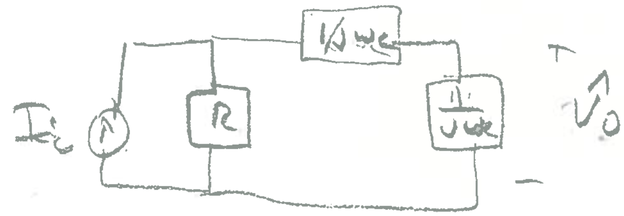
$V_o =$ $\frac{\omega L V_i}{\sqrt{(\omega L)^2 + R^2}}$ ✓

$\phi =$ $\pi/2 - \arctan(\omega L/R)$ ✓

10

(B) (*15 points*)





$i_I(t) = I_i \cos(\omega t)$

$$Z(j\omega) = R \,\|\, \left(\frac{1}{j\omega c} + \frac{1}{j\omega c}\right) = R \,\|\, \frac{2}{j\omega c}$$

$$\Rightarrow Z(j\omega) = \frac{2R/j\omega c}{R + \frac{2}{j\omega c}} = \frac{2R}{j\omega c R + 2}$$

By the current divider relation, the current through the capacitor branch is $\frac{I_i R}{R + 2/j\omega c} = \frac{j\omega c I_i R}{j\omega c R + 2}$

So the voltage through the output cap. is

$$\hat{V}_o = \frac{1}{j\omega c}\left[\frac{j\omega c I_i R}{j\omega c R + 2}\right] \Rightarrow H(j\omega) = \frac{\hat{V}_o}{I_i} = \frac{R}{j\omega c R + 2}$$

$$\hat{V}_o = \frac{I_i R}{\sqrt{4 + (\omega R c)^2}} e^{j\arctan\left(\frac{\omega R c}{2}\right)} \Rightarrow v_o(t) = \frac{I_i R}{\sqrt{4 + (\omega R c)^2}} \cos(\omega t + \varphi),$$

$$\varphi = -\arctan\left(\frac{\omega R c}{2}\right)$$

$Z(j\omega) = \frac{\hat{V}_i}{I_i} = \frac{2R}{j\omega c R + 2}$ ✓

$H(j\omega) = \frac{\hat{V}_o}{I_i} = \frac{R}{j\omega c R + 2}$ ✓

$V_o = \frac{I_i R}{\sqrt{4 + (\omega R c)^2}}$ ✓

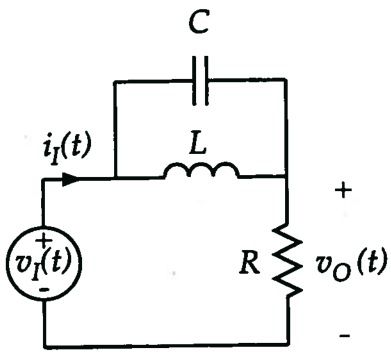
$\phi = -\arctan\left(\frac{\omega R c}{2}\right)$ ✓

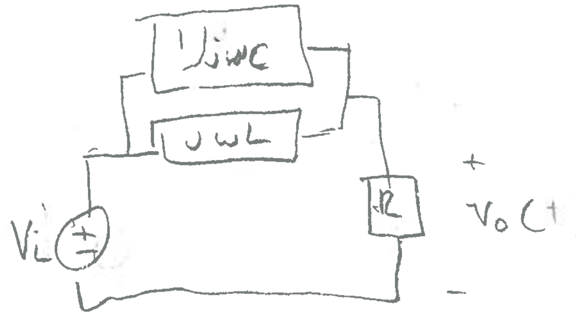
15

$$\frac{\frac{1}{j\omega C} \cdot j\omega L}{\frac{1}{j\omega C} + j\omega L} = \frac{j\omega L}{1 + (j\omega)^2 LC}$$

Name: Dan Portz

(C) *(15 points)*





$v_I(t) = V_i \cos(\omega t)$

$$Z(j\omega) = R + \left(\frac{1}{j\omega C} \parallel j\omega L\right)$$

$$\Rightarrow Z(j\omega) = R + \frac{\frac{j\omega L}{j\omega C}}{1/j\omega C + j\omega L} = R + \frac{j\omega L}{1 + (j\omega)^2 LC}$$

$$H(j\omega) = \frac{\hat{V}_o}{V_i} = \frac{\hat{I}_i R}{V_i} = \frac{R}{Z(j\omega)} = \frac{R}{R + \frac{j\omega L}{1+(j\omega)^2 LC}} = \frac{R}{R + \frac{j\omega L}{1-\omega^2 LC}}$$

$$\hat{V}_o = \frac{R}{\sqrt{R^2 + \frac{(\omega L)^2}{(1-\omega^2 LC)^2}}} e^{j \arctan\left(\frac{\omega L}{R - \omega^2 RLC}\right)}$$

$$v_o(t) = \frac{R}{\sqrt{R^2 + \frac{(\omega L)^2}{(1-\omega^2 LC)^2}}} \cos(\omega t + \varphi), \quad \varphi = -\arctan\left(\frac{\omega L}{R(1-\omega^2 LC)}\right)$$

$$Z(j\omega) = \frac{V_i}{\hat{I}_i} = R + \frac{j\omega L}{1-\omega^2 LC}$$ ✓

$$H(j\omega) = \frac{\hat{V}_o}{V_i} = \frac{R}{R + \frac{j\omega L}{1-\omega^2 LC}}$$ ✓

$$V_o = \frac{R V_i}{\sqrt{R^2 + \frac{(\omega L)^2}{(1-\omega^2 LC)^2}}}$$ ✓

$$\phi = -\arctan\left(\frac{\omega L}{R - \omega^2 RLC}\right) + (\ )$$

-2

13