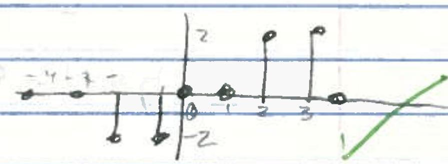
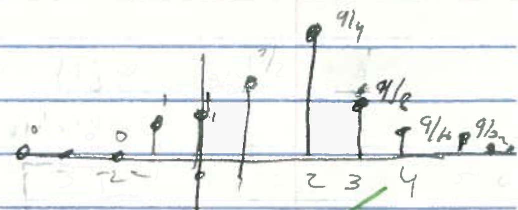
Dan Ports
6.003 PS 2

1. a. $y[n] = \begin{cases} 0, & n<0 \\ 1/2, & n=0 \\ 1, & n=1 \\ 3/2, & n=2 \\ 2, & n \geq 3 \end{cases}$

b. $y[n] = \begin{cases} -2, & n=-1, n=-2 \\ 2, & n=2, n=3 \\ 0 & \text{otherwise} \end{cases}$



c. $y[n] = \begin{cases} 0, & n<-1 \\ 1, & n=-1,0 \\ 3/2, & n=1 \\ (9/4)^{n-2}, & n \geq 2 \end{cases}$



2. a. i. $w[n] = \sum_{i=0}^{n} (-1/2)^i = \frac{(1-(-1/2)^{n+1})}{1-(-1/2)} =$

$= 2/3 \left(1-(-1/2)^{n+1}\right) u[n]$

ii. $y[n] = h_2[n] * w[n]$

$= 2/3 \left(1-(-1/2)^{n+1}\right) u[n] + 1/3 \left(1-(-1/2)^{n-1+1}\right) u[n-1]$

$= 2/3\, u[n] + 1/3\, u[n-1] - 1/3 \cdot (-1/2)^n \left(2 \cdot -1/2\, u[n] + 1\, u[n-1]\right)$

$= 2/3\, u[n] + 1/3\, u[n-1] + 1/3\, u[n] - 1/3\, u[n-1]$

$= u[n]$

b. i. $g[n] = h_2[n] * h_1[n] = (-1/2)^n u[n] + \frac{1}{2}(-1/2)^{n-1} u[n-1]$

$= \delta[n]$

ii. $y[n] = g[n] * x[n] = \delta[n] * u[n]$

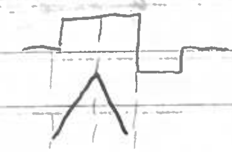
$= u[n]$

3a. We reverse the signal $h(t)$, but this has no effect because $h(-t) = h(t)$. $y(t)$ will be zero when $x(n)$ and $h(t-n)$ do not overlap, or when $h(n-t)$ overlaps equal positive and negative parts of $h(n)$.

$t \leq -3$ $t \geq 2$ $t = 0$

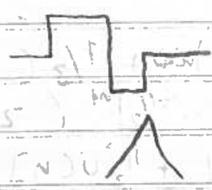
$\boxed{y(t) = 0 \text{ for } t \leq -3,\ t \geq 2,\ t = 0.}$

b. $y(t)$ has its largest possible value when $h(t-\tau)$ overlaps entirely with a positive part of $x(\tau)$.



so $y_{max} = \boxed{y(-1) = 8}$

c. Half of $h(t-\tau)$ overlaps w/ $x(\tau)$ at $t = +1$.



$y(+1) = -2 \cdot \frac{1}{2} \cdot 4 = \boxed{-4}$

4. a.

$$x(t) = e^{2(t-2)} u(-t+2)$$

$$h(t) = 2e^{-t} u(t-1)$$

$$y(t) = \int_{-\infty}^{\infty} e^{2(\tau-2)} u(-\tau+2)\, 2e^{-(t-\tau)} u(t-\tau-1)\, d\tau$$

$$= \int_{-\infty}^{2} e^{2(\tau-2)}\, 2e^{-(t-\tau)} u(t-\tau-1)\, d\tau$$

For $t>3$, $y(t) = \int_{-\infty}^{2} e^{2(\tau-2)} \cdot 2e^{-(t-\tau)}\, d\tau$

$$= \int_{-\infty}^{2} e^{2\tau} e^{-4} \cdot 2e^{-t} e^{\tau}\, d\tau$$

$$= 2e^{-t} e^{-4} \int_{-\infty}^{2} e^{3\tau}\, d\tau$$

$$= 2e^{-t} \cdot e^{-4} \left.\frac{e^{3\tau}}{3}\right|_{-\infty}^{2} = \frac{2}{3} e^{-t} e^{-4} e^{6}$$

$$= \frac{2}{3} e^{-t+2}$$

For $t<3$, $y(t) = \int_{-\infty}^{t-1} e^{2(\tau-2)} \cdot 2e^{-(t-\tau)}\, d\tau$

$$= \int_{-\infty}^{t-1} e^{2\tau} e^{-4} \cdot 2e^{-t} e^{\tau}\, d\tau$$

$$= 2e^{-t} e^{-4} \int_{-\infty}^{t-1} e^{3\tau}\, d\tau$$

$$= 2e^{-t} e^{-4} \left.\frac{e^{3\tau}}{3}\right|_{-\infty}^{t-1}$$

$$= 2e^{-t} e^{-4} \frac{e^{3(t-1)}}{3}$$

$$= \frac{2}{3} e^{2t-7}$$

$$\boxed{y(t) = \begin{cases} \frac{2}{3} e^{-t+2} & \text{for } t \geq 3 \\ \frac{2}{3} e^{2t-7} & \text{for } t < 3 \end{cases}}$$

b $x(t) = u(t+1)$

$h(t) = (1 - t/2)[u(t) - u(t-2)]$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} [u(\tau) - u(\tau-2)](1-\tau/2)\, u(t-\tau+1)\, d\tau$$

$$= \int_0^2 (1-\tau/2)\, u(t-\tau+1)\, d\tau$$

For $t < -1$, $y(t) = 0$

For $-1 \le t \le 1$

$$y(t) = \int_0^{t+1} 1 - \tau/2 \, d\tau$$

$$= \left[\tau - \tau^2/4\right]\Big|_0^{t+1}$$

$$= (t+1) - (t+1)^2/4 = t+1 - \frac{t^2}{4} - \frac{2t}{4} - \frac{1}{4}$$

$$= -t^2/4 + t/2 + 3/4$$

For $t > 1$, $y(t) = \int_0^2 1 - \tau/2 \, dt$

$$= 1$$

$$y(t) = \begin{cases} 0, & t < -1 \\ -t^2/4 + t/2 + 3/4, & -1 \le t \le 1 \\ 1, & t > 1 \end{cases}$$

5. a Substituting the proposed soln for $y[n]$

$$A(-2)^n + B(1/2)^n + 3/2\,[A(-2)^{n-1} + B(1/2)^{n-1}]$$
$$- [A(-2)^{n-2} + B(1/2)^{n}] = 0$$
$$\Rightarrow A(-2)^{n-2}\,[-1 + 3/2 \cdot -2 + 1 \cdot (-2)^2]$$
$$+ B(1/2)^{n-2}\,[-1 + 3/2 \cdot 1/2 + 1\,(1/2)^2] = 0$$
$$\Rightarrow A(-2)^{n-2}\,[-1 - 3 + 4] + B[-1 + 3/4 + 1/4] = 0$$
$$\Rightarrow A[0] + B[0] = 0$$, which is always true.

b. We expect a soln of form
$$A(-2)^n u[n] + B(1/2)^n u[n]$$
Substituting, at $n=0$ we have
$$A + B + 0 + 0 = 1$$
At $n=1$, $-2A + 1/2\,B + B/2 = 0$
$$\Rightarrow A = 4/5, \quad B = 1/5$$

$$y[n] = 1/5\,(-2)^n u[n] + 4/5\,(1/2)^n u[n]$$

c. Expect a soln of form
$$A(-2)^n u[-n-1] + B(1/2)^n u[n]$$

Substituting, at $n=0$ we have
$$B + 3/2 \cdot A \cdot (-2)^{-1} - A\,(-2)^{-2} = 1$$
At $n=1$,
$$1/2\,B + 3/2\,B - A \cdot (-2)^{-1}$$
$$\Rightarrow A = -4/5, \quad B = 1/5$$

$$y[n] = -4/5\,(-2)^n u[-n-1] + 1/5\,(1/2)^n u[n]$$

6. a. Substituting the given $y(t)$,

$$-4Ae^{-2t} + \tfrac{1}{4}Be^{1/2 t} + \tfrac{3}{2}\left(-2Ae^{-2t} + \tfrac{1}{2}Be^{1/2 t}\right)$$
$$-\left(Ae^{-2t} + Be^{1/2 t}\right) = 0$$
$$\Rightarrow Ae^{-2t}\left(4 + \tfrac{3}{2}\cdot -2 - 1\right)$$
$$+ Be^{1/2 t}\left(\tfrac{1}{4} + \tfrac{3}{2}\cdot\tfrac{1}{2} - 1\right) = 0$$
$$\Rightarrow Ae^{-2t}(0) + Be^{1/2 t}(0) = 0$$ which is always true ✓

b. We expect a sol'n of form
$Ae^{-2t}u(t) + Be^{1/2 t}u(t) + C\delta(t)$, but
$C = 0$ by singularity-matching.

$$\tfrac{3}{2}\frac{dy}{dt} = -3Ae^{-2t}u(t) + \tfrac{3}{2}A\delta(t) + \tfrac{3}{4}Be^{-2t}u(t) + \tfrac{3}{2}B\delta(t)$$
$$+ \frac{d^2y}{dt^2} = 4Ae^{-2t}u(t) - 2A\delta(t) + A\dot{\delta}(t)$$
$$+ \tfrac{1}{4}Be^{-2t}u(t) + \tfrac{1}{2}B\delta(t) + B\dot{\delta}(t)$$
$$+ -y(t) = -Ae^{-2t}u(t) - Be^{1/2 t}u(t)$$
$$= \left(\tfrac{3}{2}A - 2A + \tfrac{3}{2}B + \tfrac{1}{2}B\right)\delta(t) + (A+B)\dot{\delta}(t) = \delta(t)$$

$A + B = 0$, $-\tfrac{1}{2}A + 2B = 1$
$A = -2/5$, $B = 2/5$

$$\boxed{y(t) = -\tfrac{2}{5}e^{-2t}u(t) + \tfrac{2}{5}e^{1/2 t}u(t)}$$ ✓

c. We expect a sol'n of form
$Ae^{-2t}u(t) + Be^{1/2 t}u(-t) + C\delta(t)$!
$C = 0$ by singularity matching.

$$-y(t) = -Ae^{-2t}u(t) - Be^{1/2 t}u(-t)$$
$$\tfrac{3}{2}\dot{y}(t) = -3Ae^{-2t}u(t) + \tfrac{3}{2}A\delta(t)$$
$$+ \tfrac{3}{4}Be^{-2t}u(-t) - \tfrac{3}{2}B\delta(-t)$$
$$\ddot{y}(t) = 4Ae^{-2t}u(t) - 2A\delta(t) + A\dot{\delta}(t)$$

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$A - B = 0, \quad -\tfrac{1}{2}A - 2B = 1$

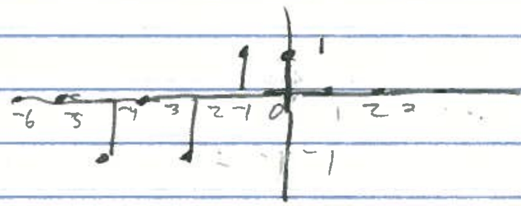
$\Rightarrow A = B = 2/5$

almost !! :)

$$\boxed{y(t) = \tfrac{2}{5} e^{-2t} u(t) + \tfrac{2}{5} e^{1/2 t} u(-t)}$$

7.a. The matched filter is the system that convolves its input with the time-reversal of $x[n]$. The unit sample response is $\delta[n] * x[-n] = x[-n]$ ✓

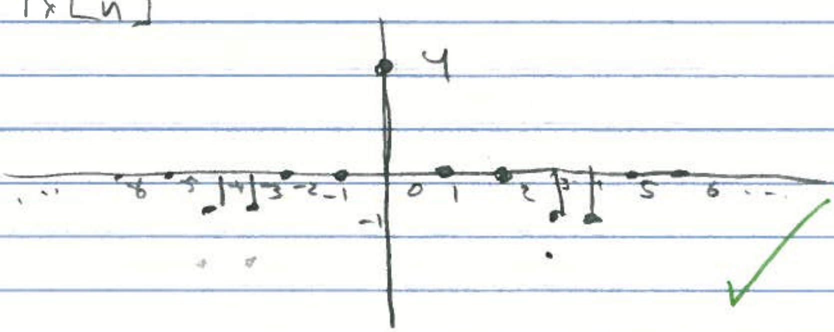
b. $h[n] = x[-n] = \delta[n] + \delta[-n-1] - \delta[-n-2] - \delta[n-4]$



✓

c. Performing graphically the convolution of $x[n]$ and $x[-n]$,

$r_x[n]$



✓

$$r_x[n] = \begin{cases} 4, & n = 0 \\ -1, & n = -3, -4, 3, 4 \\ 0 & \text{elsewhere} \end{cases}$$

just print out your code next time

handwritten code = no grade

8. The noise can be removed with the following set of commands

```
>> c = conv(noise_easy, pattern(end:-1:1));
>> d = diff(c);
>> i = find(d>1);
>> j = i-18;
>> z = remove_noise(noise_easy, pattern, j);
>> soundsc(z, rate);
```

This correctly identifies and removes all 1500 noise points.