(3)

Dan Portz

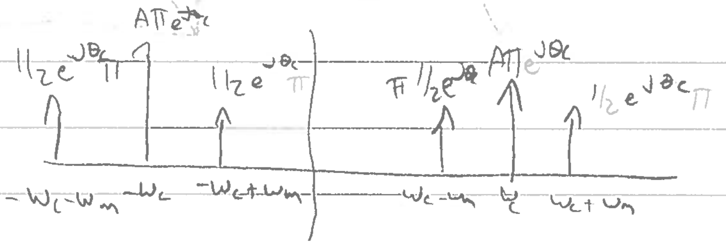
6.003 PS 7

1. 8.27

a. $y(t) = [A + \cos(\omega_m t)] \cdot \cos(\omega_c t + \theta_c)$

$$\Rightarrow Y(j\omega) = \frac{1}{2\pi}\left[2\pi A\,\delta(\omega) + \pi\,\delta(\omega-\omega_m) + \pi\,\delta(\omega+\omega_m)\right] * \left[\pi\,\delta(\omega-\omega_c)e^{j\theta_c} + \pi\,\delta(\omega-\omega_c)e^{-j\theta_c}\right]$$

$$= \pi e^{j\theta_c}\left[A\,\delta(\omega-\omega_c) + \tfrac{1}{2}\delta(\omega-\omega_m-\omega_c) + \tfrac{1}{2}\delta(\omega+\omega_m-\omega_c)\right] + \pi e^{-j\theta_c}\left[A\,\delta(\omega+\omega_c) + \tfrac{1}{2}\delta(\omega-\omega_m+\omega_c) + \tfrac{1}{2}\delta(\omega+\omega_m+\omega_c)\right]$$



Converting this Fourier transform into a Fourier series, we use Parseval's relation to find

$$P_y = \frac{1}{T}\sum |a_k|^2 = \frac{1}{T}2\left[\left(\frac{1}{4}\right)^2 + \left(\frac{A}{2}\right)^2 + \left(\frac{1}{4}\right)^2\right]$$

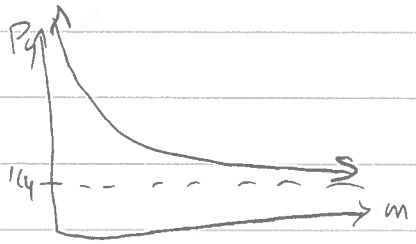
$$= \frac{A^2}{2} + \frac{1}{4} \checkmark$$

because the Fourier series coefficients are just the areas of the impulses divided by 2π, and ~~the absolute value eliminates~~ the $e^{\pm j\theta}$ phase shift.

Note that $\max\{x(t)\} = 1$, so $m = 1/A$

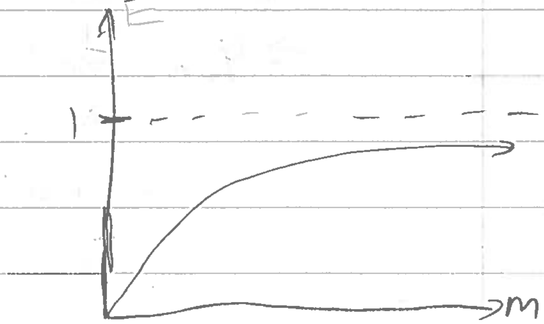
$\Rightarrow A = 1/m$

$$P_y = \frac{1}{4} + \frac{1}{2m^2} \checkmark$$



b. The total power is $\frac{1}{4} + \frac{1}{2m^2}$.
The power in the sidebands is $4 \cdot (\frac{1}{4})^2$, considering only the Fourier coefficients that come from the signal. So the efficiency is

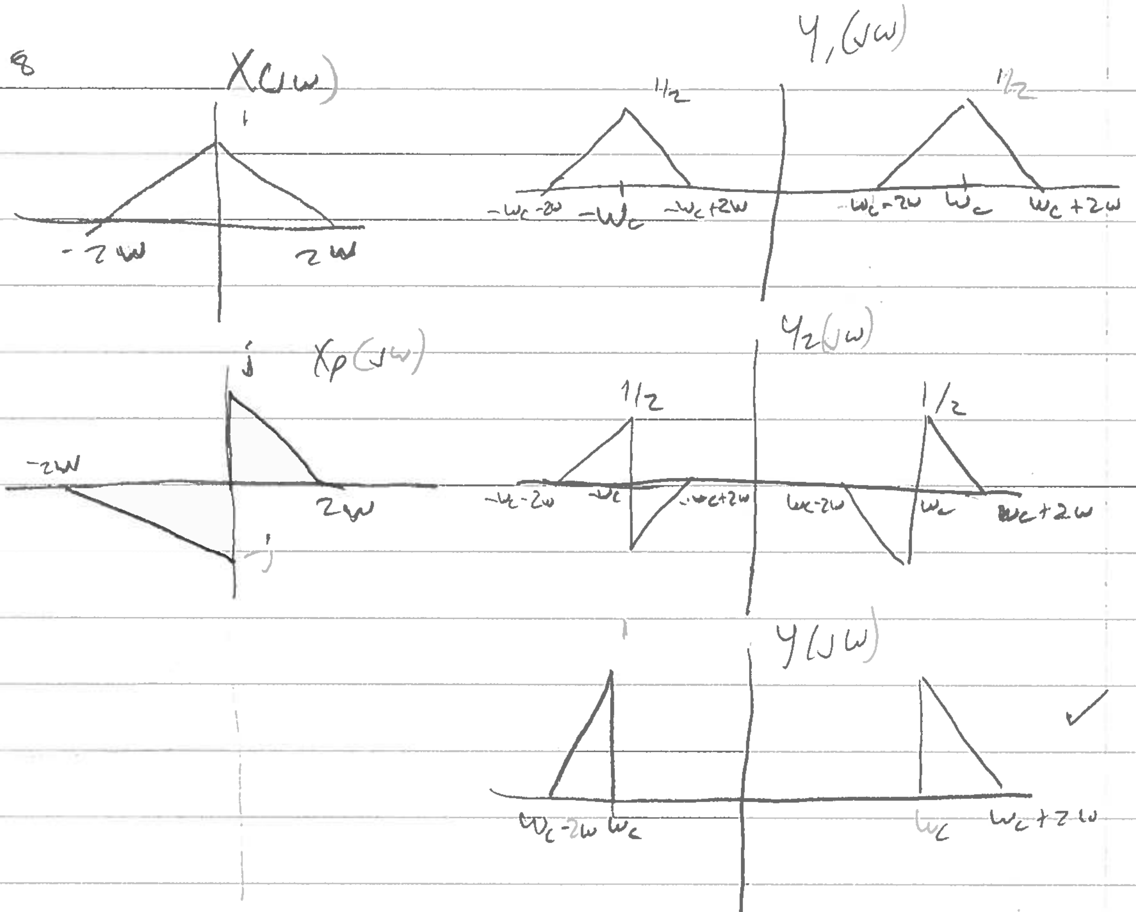
$$\frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2m^2}}$$



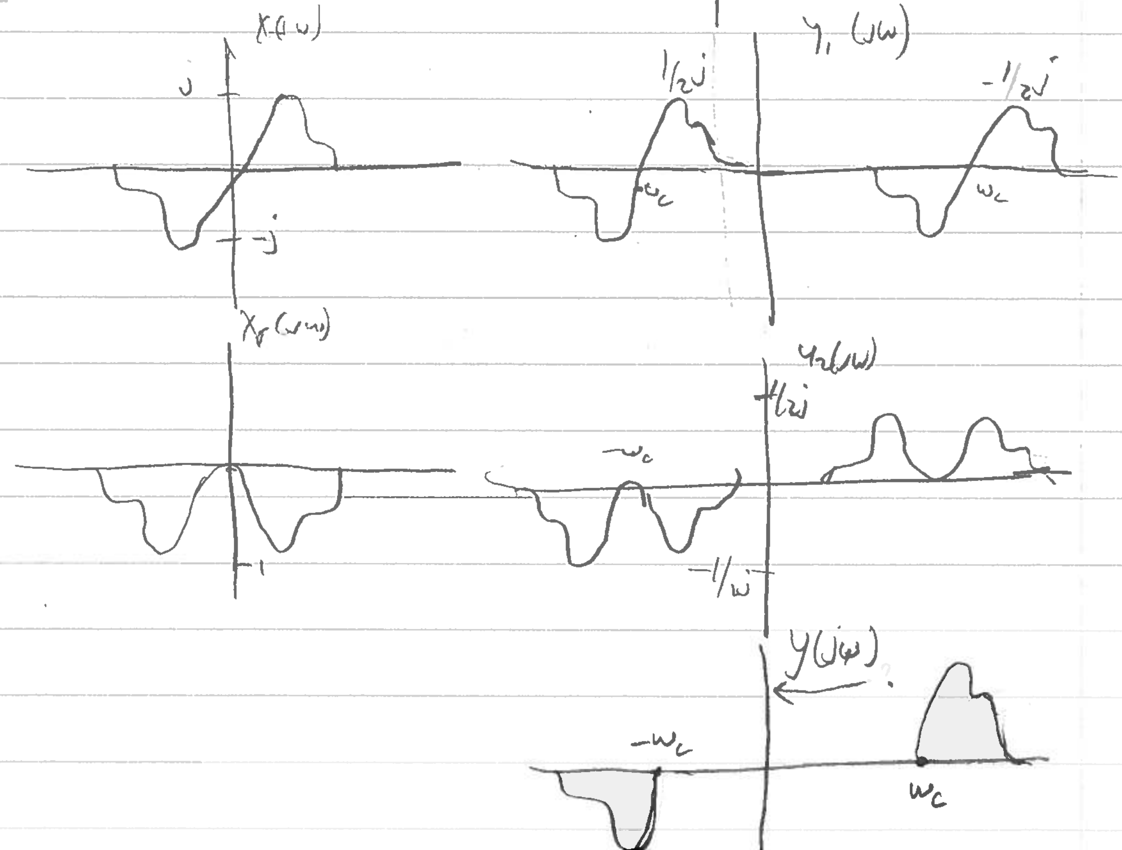
✓

2 8.28

a.

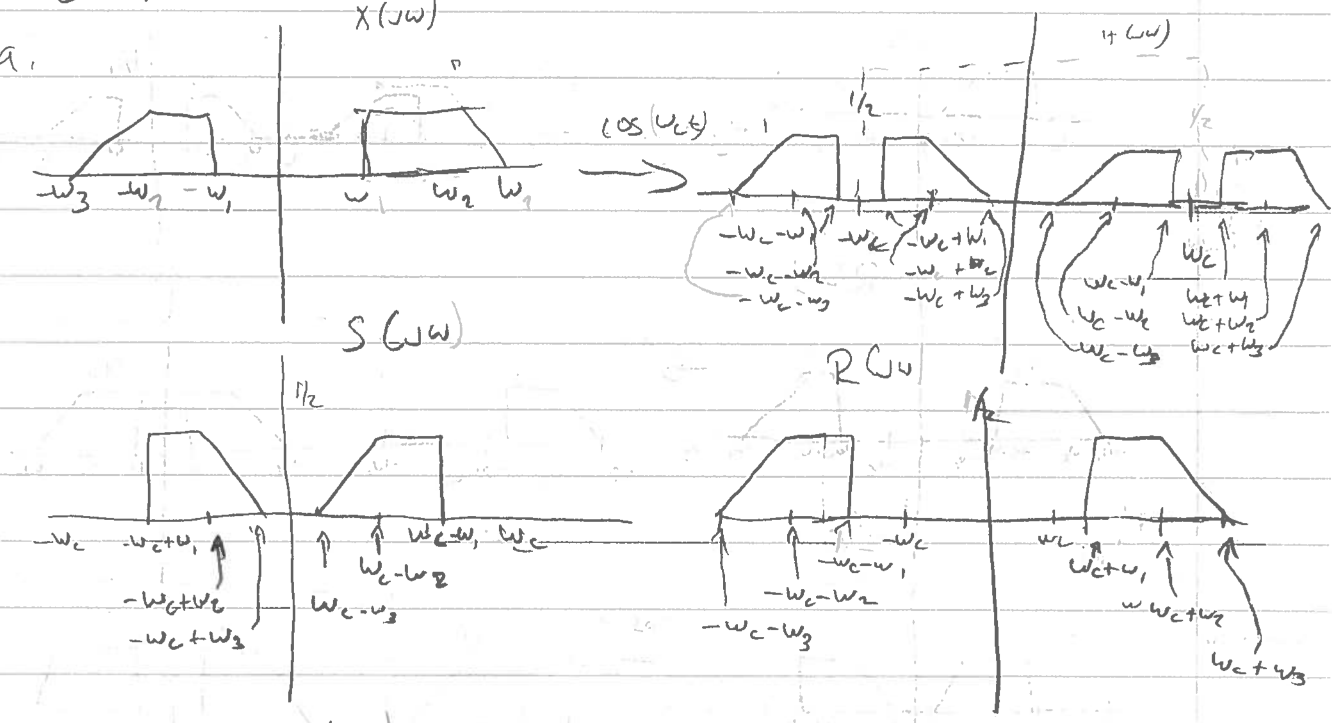


b.

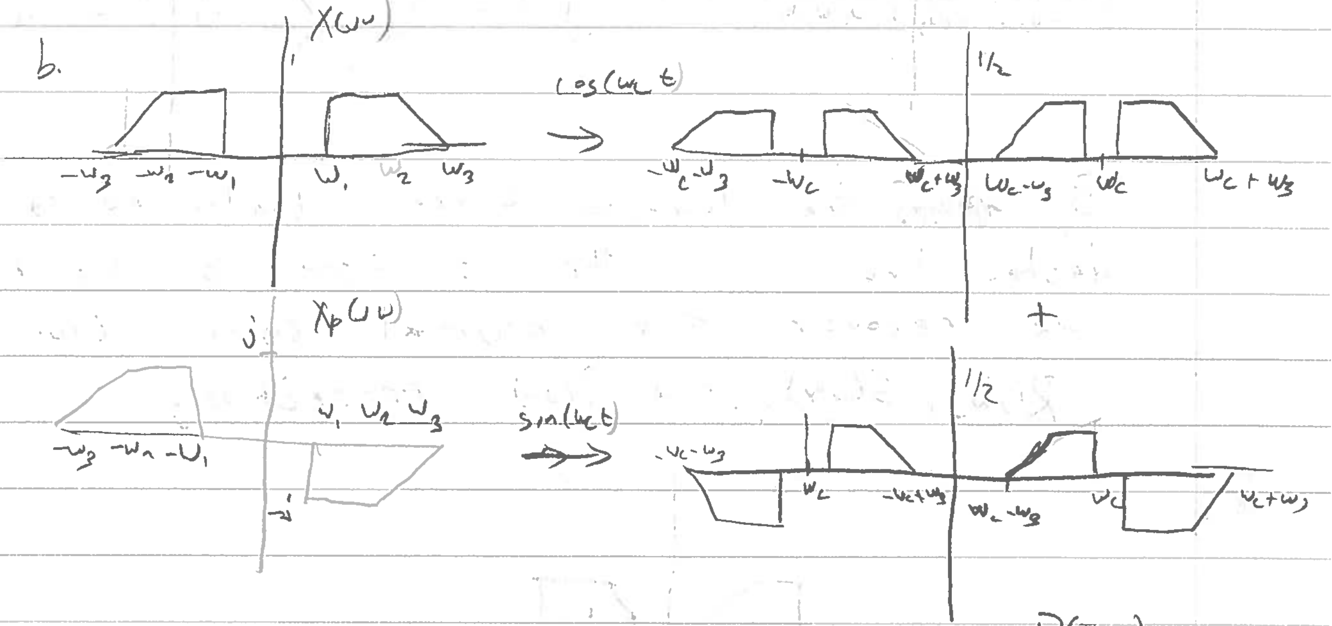


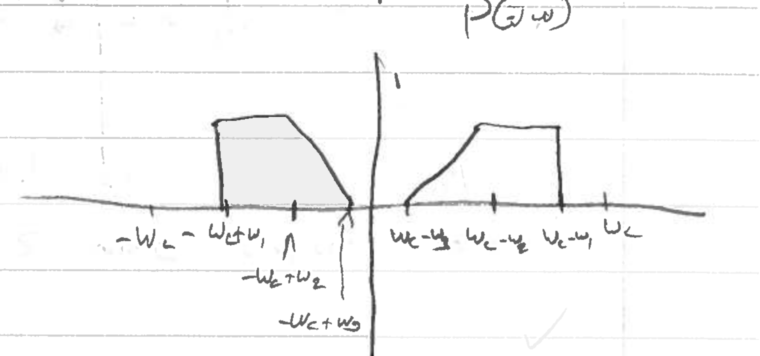
3. 8.29

a.



b.

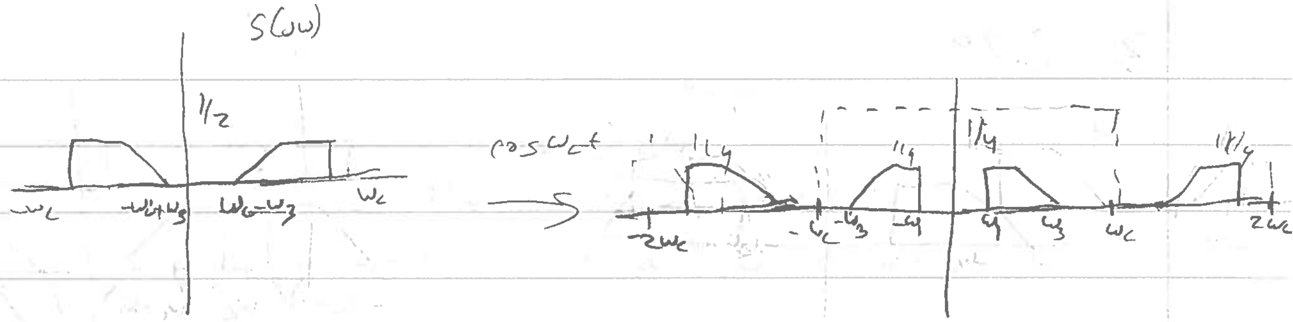


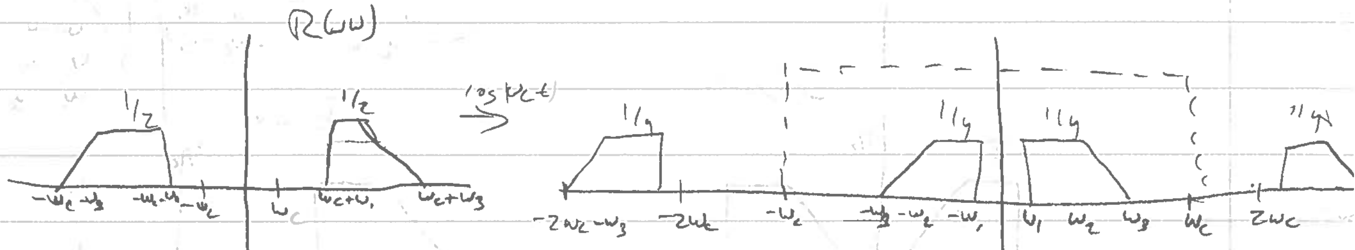


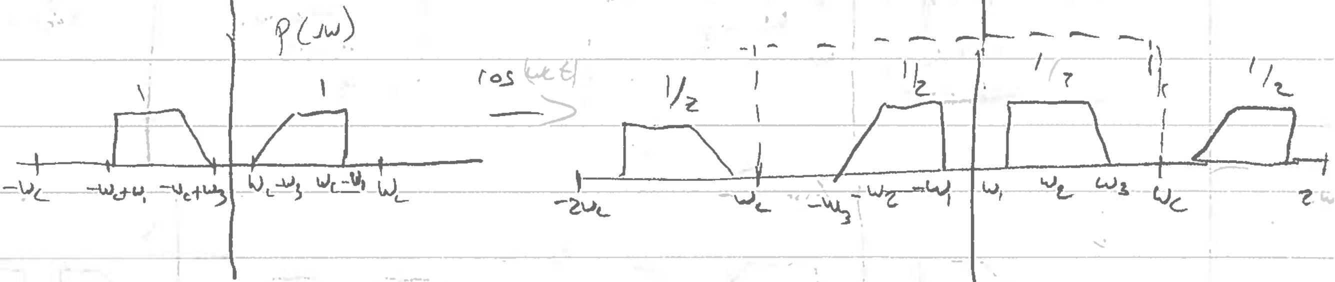
So $P(j\omega) = 2S(j\omega)$

$\Rightarrow p(t) = 2s(t)$

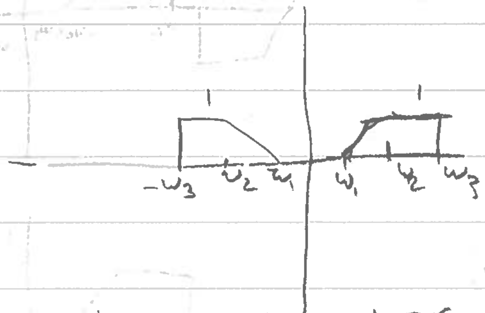
c.







By applying the low-pass filters (indicated by the dashed line) to the 3 signals on the right, we recover the original signal $X(j\omega)$ from $R(\omega)$, $S(\omega)$, and $P(j\omega)$ respectively!

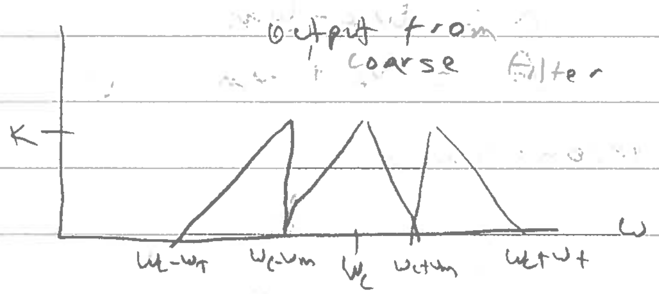


Note that the LPF in the system for $p(t)$ only needs gain 2 since $p(t) = 2s(t)$.

✓

4. 8.36.

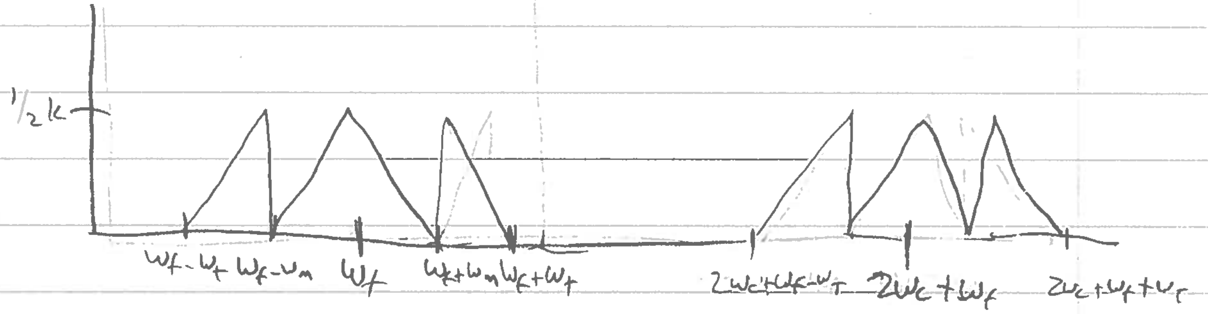
a.



call this $Y_2(j\omega)$

$$Z(j\omega) = Y_2(j\omega) * \left[\tfrac{1}{2}\delta(\omega-\omega_c-\omega_f) + \tfrac{1}{2}\delta(\omega+\omega_c+\omega_f)\right]$$

$$= \tfrac{1}{2}\left[Y_2(j(\omega-\omega_c-\omega_f)) + Y_2(j(\omega+\omega_c+\omega_f))\right]$$



and symmetric about $\omega = 0$

b. We need to ensure that there is no overlap between the modulated signal Y_1 and the other non-zero frequencies.

$$\omega_f + \omega_m < 2\omega_c + \cancel{\omega_f} - \omega_T$$

$$\omega_m < 2\omega_c - \omega_T$$

$$\boxed{\omega_T < 2\omega_c - \omega_m}$$

✓

c. We choose $G = 2/k$

$$\alpha = \omega_f - \omega_m$$

$$\beta = \omega_f + \omega_m \quad \checkmark$$

for reconstruction