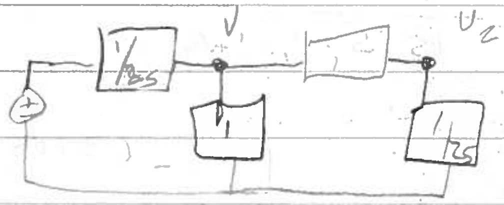
3

Dan Ports
6.003 PS 9

1. a.




$$(v_1 - v_0)\,2s + (v_1 - v_2)\,1 + v_1 = 0$$
$$(v_2 - v_1)\,1 + (v_2 - 0)\,2s = 0$$

$$\Rightarrow \quad V_1(2+2s) - V_2 = V_0\,2s$$
$$-V_1 + (1+2s)V_2 = 0$$

$$\begin{bmatrix} 2+2s & -1 \\ -1 & 1+2s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_0\,2s \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{4s^2+6s+1} \begin{bmatrix} 2s(2s+1)V_0 \\ 2s\,V_0 \end{bmatrix}$$

$$\boxed{A(s) = \frac{V_2}{V_0} = \frac{2s}{4s^2+6s+1}}$$

$$(4s^2+6s+1)V_2 = 2s\,V_0$$

$$\Rightarrow \boxed{4\frac{d^2 v_2}{dt^2} + 6\frac{d v_2}{dt} + v_2 = 2\frac{d v_0}{dt}}$$

c. $v_0(t) = u(t) \Rightarrow V_0(s) = 1/s, \quad Re(s) > 0$

$$V_2(s) = H(s)\,V_0(s) = \frac{2}{4s^2+6s+1} = \frac{1/2}{(s + 1/4(3+\sqrt{5}))(s + 1/4(3-\sqrt{5}))}$$

$$= \frac{A}{s + 1/4(3+\sqrt{5})} + \frac{B}{s + 1/4(3-\sqrt{5})}$$

$$A = \frac{1/2}{-1/4(3+\sqrt{5}) + 1/4(3-\sqrt{5})} = \frac{-1}{\sqrt{5}}$$

$$B = \frac{1/2}{-1/4(3+\sqrt{5}) + 1/4(3-\sqrt{5})} = 1/\sqrt{5}$$

$$V_2(s) = \frac{1}{\sqrt{5}} \frac{1}{s + 1/4(3-\sqrt{5})} - \frac{1}{\sqrt{5}} \frac{1}{s + 1/4(3+\sqrt{5})}$$

$V_2(t)$ is causal

$$V_2(t) = \frac{1}{\sqrt{5}} e^{-\frac{(3-\sqrt{5})}{4}t} u(t) - \frac{1}{\sqrt{5}} e^{-\frac{(3+\sqrt{5})}{4}t} u(t)$$

2. a. Sys 1: $sY(s) = X(s) \Rightarrow Y(s) = X(s)/s$

$X(s) = \mathcal{L}(1/s) = 1/s$ (ROC: $Re(s) > 0$)

$Y(s) = 1/s^2$

$\Rightarrow \boxed{y(t) = t\,u(t)}$

Sys 2: $sY(s) + aY(s) = X(s)$

$\Rightarrow Y(s) = \frac{X(s)}{s+a} = \frac{1}{s(s+a)} = \frac{1/a}{s} + \frac{-1/a}{s+a}$

$\boxed{y(t) = 1/a\,u(t) - 1/a\,e^{-at}u(t)}$

b. For sys 1:

$\boxed{\lim_{t\to\infty} y(t) = \infty}$

For sys 2:

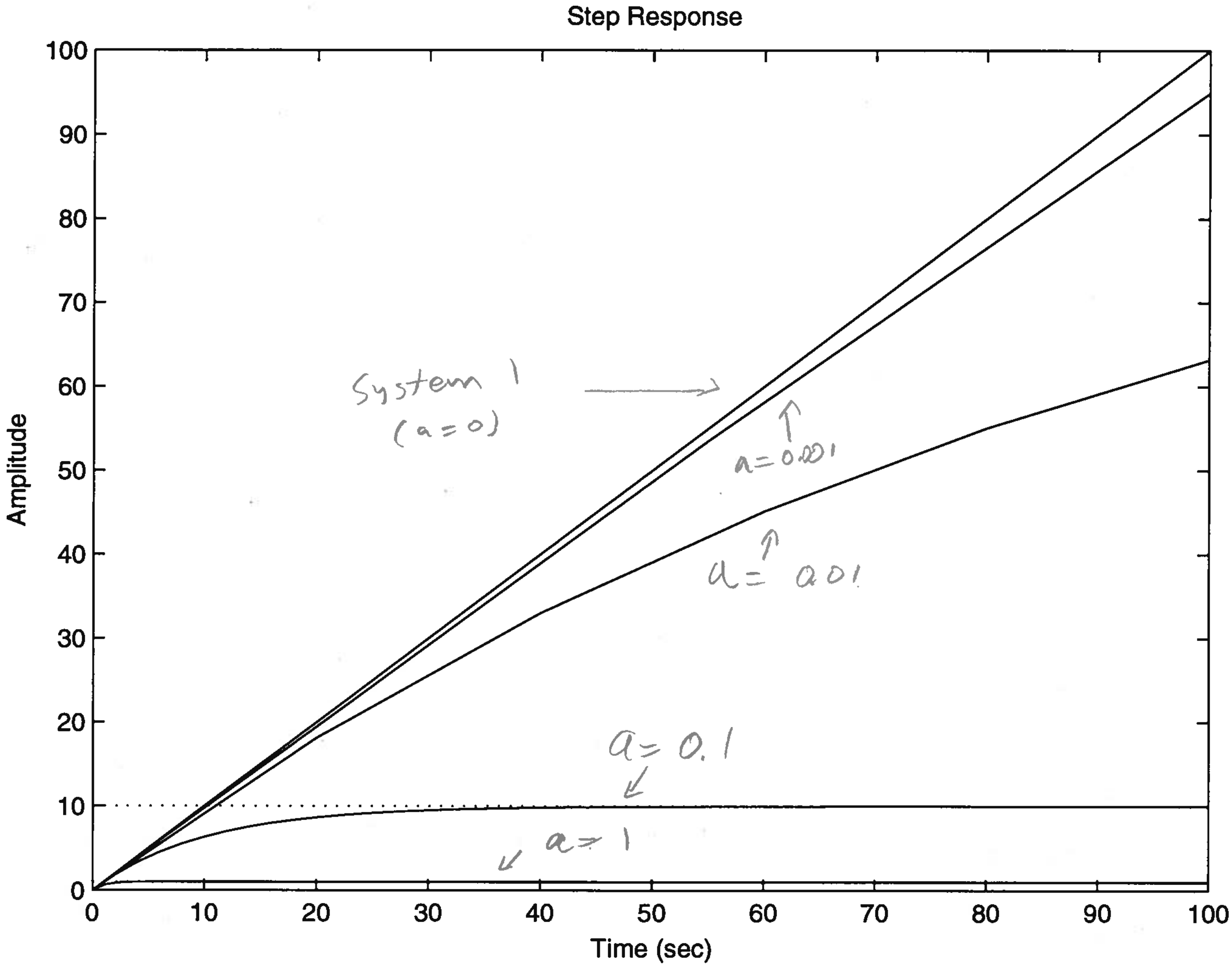
$\lim_{t\to\infty} y(t) = 1/a - 1/a\,e^{-at} = \boxed{1/a}$

c. See the attached Matlab plots. As a is decreased, the limit as $t\to\infty$ (which is $1/a$) increases toward infinity. The graph of system 2 approximates the linear curve of system 1 for a short time until the slope of the curve drops off. As $a \to 0$, the curve approximates a straight line $t\,u(t)$ for an increasingly long time, so as $a \to 0$ the response of system 2 approaches the $t\,u(t)$ response

2.1

2c

Step responses. system 1 & 2

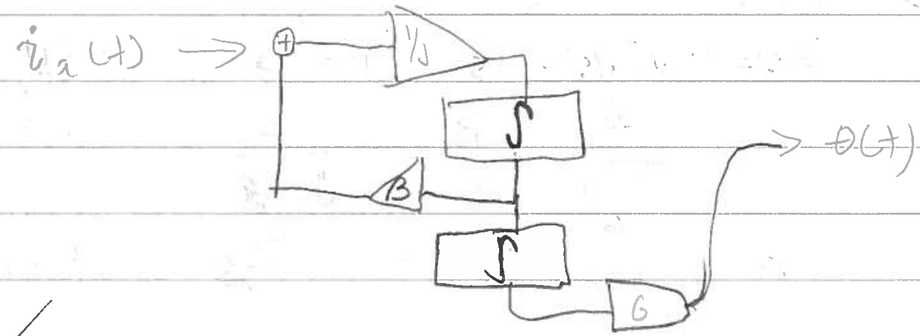


```
diary on
hold off
sys=tf([1],[1 0])

Transfer function:
1
-
s

step(sys)
sys=tf([1],[1 1])

Transfer function:
  1
-----
s + 1

hold on
step(sys)
sys=tf([1],[1 .1])

Transfer function:
   1
-------
s + 0.1

step(sys)
sys=tf([1],[1 .01])

Transfer function:
   1
--------
s + 0.01

step(sys)
sys=tf([1],[1 .001])

Transfer function:
    1
---------
s + 0.001

step(sys)
axis([0 100 0 100])
diary off
```

3. a. $\tau_m(t) - \tau_F(t) = J\frac{d^2\theta(t)}{dt^2}$

$\Rightarrow G\, i_a(t) - \beta \frac{d\theta(t)}{dt} = J \frac{d^2\theta(t)}{dt^2}$

$\Rightarrow G\, i_a(t) = J\frac{d^2\theta(t)}{dt^2} + \beta\frac{d\theta(t)}{dt}$




b. $G I_a(s) = J s^2 \Theta(s) + \beta s \Theta(s)$

$$\boxed{H(s) = \frac{\Theta(s)}{I_a(s)} = \frac{G}{Js^2+\beta s}}$$

c. $H(s) = \frac{1}{s^2+2s}$ $\qquad I_a(s) = 1/s$, $Re(s) > 0$

$\Theta(s) = \frac{1}{s(s^2+2s)} = \frac{1}{s^2(s+2)}$

$= \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+2}$

$A s(s+2) + B(s^2)(s+2) + C s^3 = s$

$\Rightarrow A(s^2+2s) + B(s^3+2s^2) + C s^3 = s$

$A = 1/2 \qquad A+2B = 0 \Rightarrow B = -1/4 \qquad C = -B = 1/4$

$\Theta(s) = \frac{1/2}{s^2} - \frac{1/4}{s} + \frac{1/4}{s+2}$

$$\boxed{\theta(t) = \tfrac{1}{2} t\, u(t) - \tfrac{1}{4} u(t) + \tfrac{1}{4} e^{-2t} u(t)}$$

d. No. $\lim_{t \to \infty} \theta(t) = \infty$ because of the $1/\beta\, t\, u(t)$ term.

e. $I_a(s) = \frac{1}{s-1}$

$\theta(s) = H(s) I_a(s) = \frac{1}{s-1}\left(\frac{6}{Js^2+\beta s}\right) = \frac{6}{(s-1)\, s(Js+\beta)}$

$= \frac{A}{s-1} + \frac{B}{s} + \frac{C}{Js+\beta}$

$A = \frac{6}{J+\beta}$, $B = \frac{6}{-\beta}$, $C = \frac{6}{(-\beta/J - 1)(-\beta/J)}$

$= \frac{6J^2}{\beta^2+\beta J}$

$\theta(s) = \frac{\frac{6}{J+1}}{s-1} - \frac{6/\beta}{s} + \frac{6J^2}{\beta^2+\beta J}\left(\frac{1}{Js+\beta}\right)$

$$\boxed{\theta(t) = \underbrace{\frac{6}{J+\beta} e^{t} u(t) - \frac{6}{\beta} u(t)}_{\text{Steady-state response}} + \underbrace{\frac{6J}{\beta^2+\beta J} e^{-\beta/J\, t} u(t)}_{\text{transient}}}$$

f. $6 I_a(s) = Js^2 \theta(s) + \beta s \theta(s)$

$\omega(s) = s\theta(s)$

$\Rightarrow 6 I_a(s) = Js\, W(s) + \beta W(s)$

$$\boxed{H_w(s) = \frac{W(s)}{I_a(s)} = \frac{6}{Js+\beta}}$$

g. Applying the final value thm, the steady state response $\lim_{t\to\infty} \theta(t) = \lim_{s\to 0} s\theta(s)$

$= \lim_{s\to 0} s H_w(s) \frac{1}{s} = \lim_{s\to 0} H_w(s) = \lim_{s\to 0} \frac{6}{Js+\beta}$

$= \boxed{6/\beta}$. It is acceptable to use the final value thm because $H_w(s) \frac{1}{s}$

$= \frac{6}{(Js+\beta)s}$, which has no poles to the right of the $j\omega$ axis, so

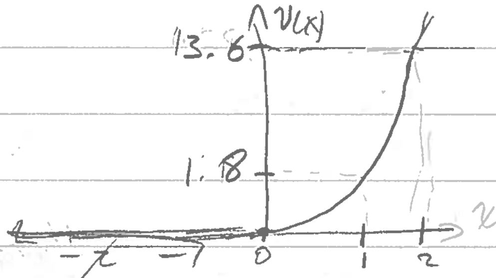
4.a $\frac{d^2 v(x)}{dx^2} - 4\, v(x) = \delta(x)$

$$s^2 V(s) - 4\, V(s) = 1$$

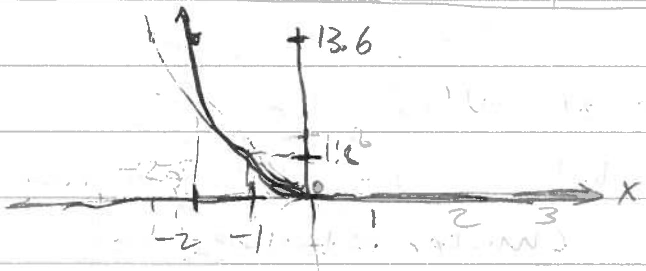
$$V(s) = \frac{1}{(s^2-4)} = \frac{1}{(s+2)(s-2)} = \frac{A}{s+2} + \frac{B}{s-2}$$

$$A = -1/4 \qquad B = 1/4$$

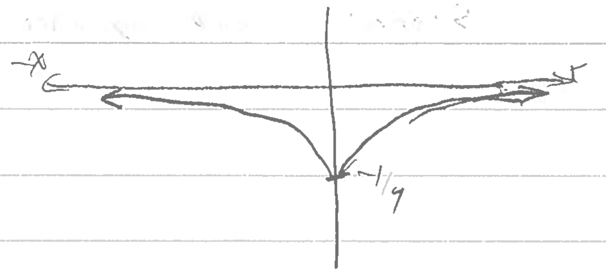
$$\boxed{v(x) = -1/4\, e^{-2x} u(x) + 1/4\, e^{2x} u(x)}$$ causal




b $$\boxed{v(x) = 1/4\, e^{-2x} u(-x) - 1/4\, e^{2x} u(-x)}$$ anticausal




c $$\boxed{v(x) = -1/4\, e^{-2x} u(x) - 1/4\, e^{2x} u(-x)}$$



d. If a nerve is stimulated in the middle, we expect the signal to travel in both directions. We also do not expect the signal to have infinite magnitude when it reaches the endpoints. This is consistent with the stable response.

e. Suppose $\sigma_n = 1$ and $\sigma_e$ varies:

$$\frac{d^2 v(x)}{dx^2} - \sigma_e v(x) = \delta(x)$$

$$s^2 V(s) - \sigma_e V(s) = 1$$

$$V(s) = \frac{1}{s^2 - \sigma_e} = \frac{1}{(s-\sqrt{\sigma_e})(s+\sqrt{\sigma_e})} = \frac{1/2\sqrt{\sigma_e}}{s-\sqrt{\sigma_e}} - \frac{1/2\sqrt{\sigma_e}}{s+\sqrt{\sigma_e}}$$

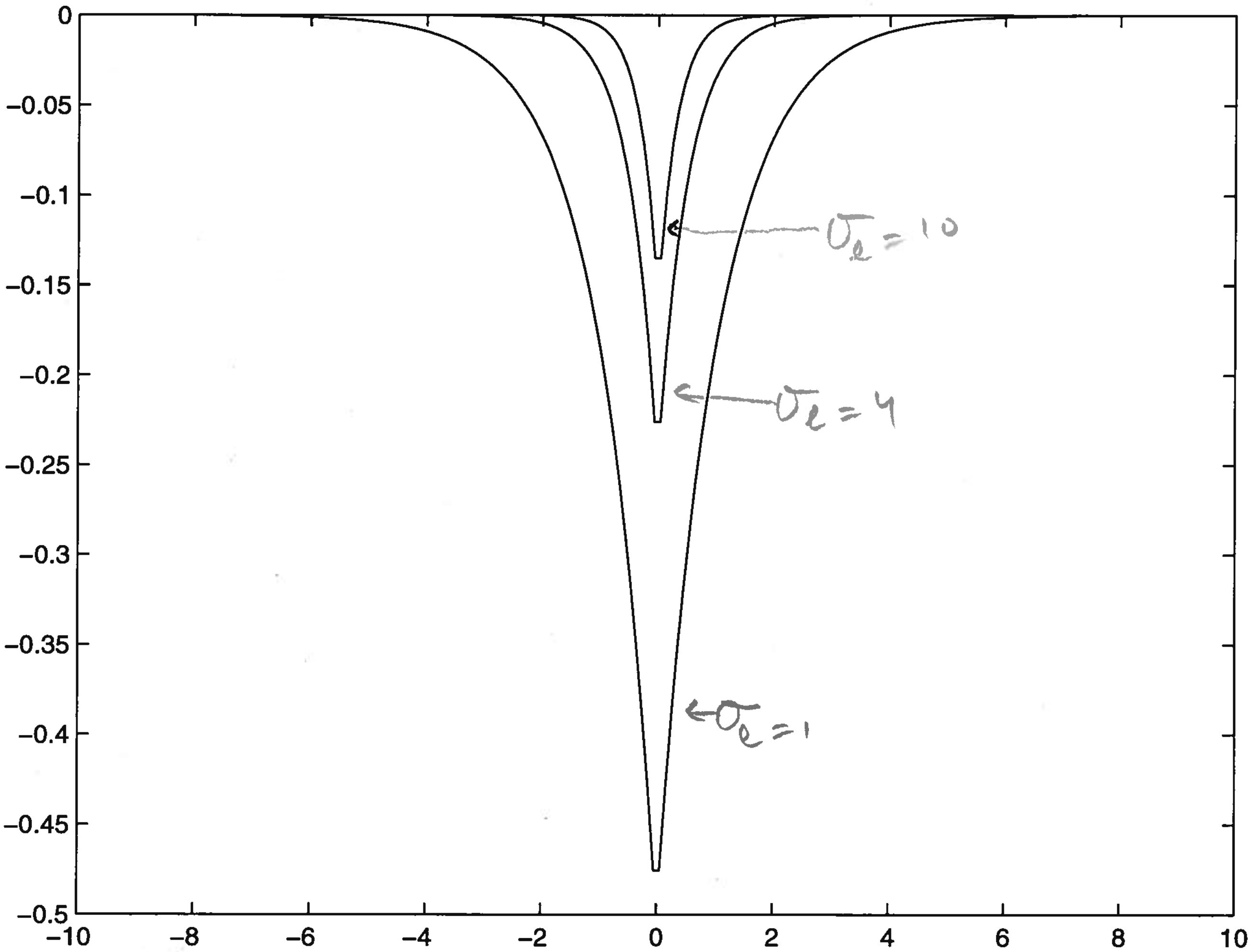
$$\xrightarrow{\mathcal{L}^{-1}} \frac{1}{2\sqrt{\sigma_e}}\left[-e^{-\sqrt{\sigma_e}x} u(x) - e^{\sqrt{\sigma_e}x} u(-x)\right]$$

A plot of $v(x)$ for $\sigma_n = 1$ and $\sigma_e = 1, 4, 10$ is attached. It is evident from the plot that smaller values of $\sigma_e$ relative to $\sigma_n$ result in a larger conducted signal. Thus, raising the temperature worsens signal propagation.

4.

4e

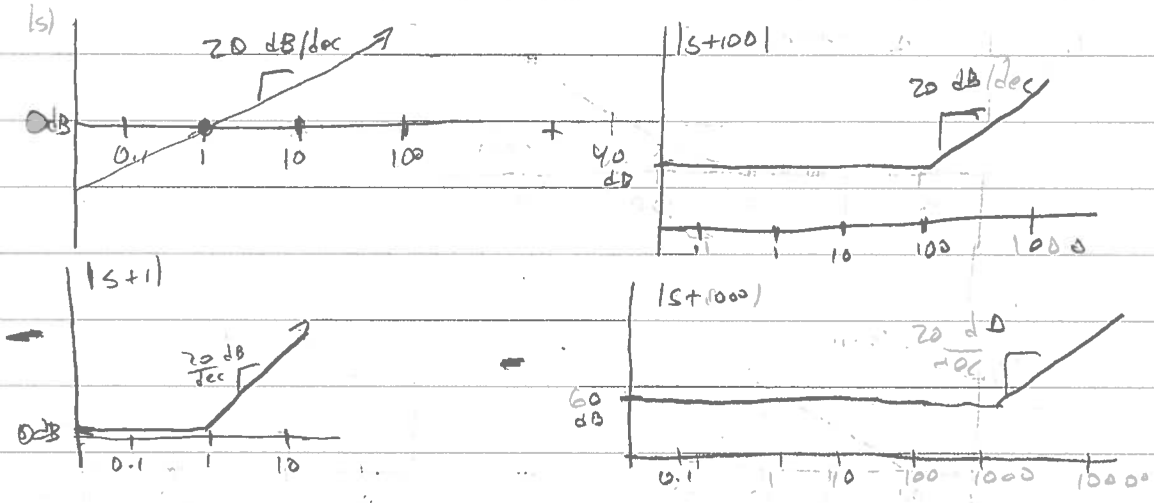
$\sigma_n = 1$ throughout.

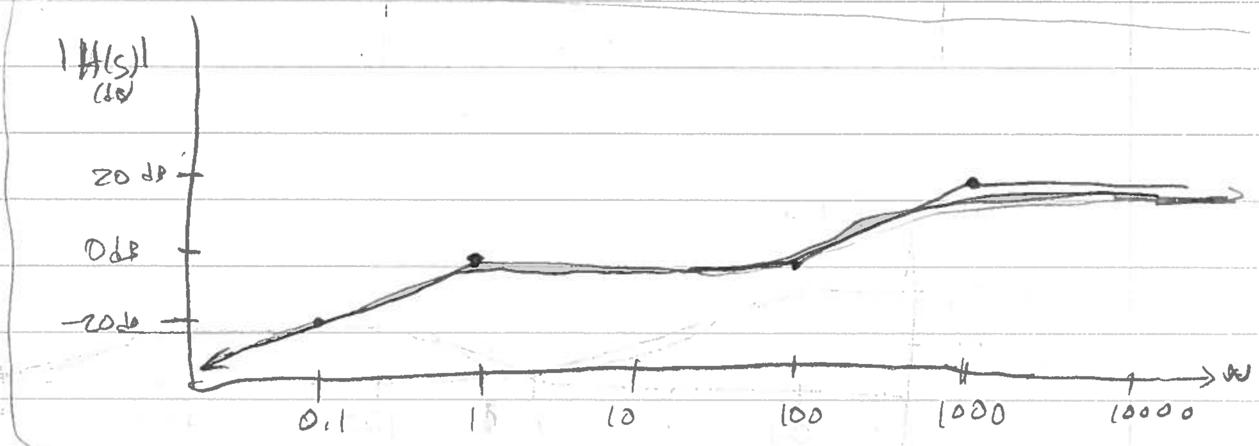


```
t=linspace(-10,10,200);
k=1

k =

     1

x(1:100)=(-1/(2*sqrt(k)))*exp(sqrt(k)*t(1:100));
x(101:200)=(-1/(2*sqrt(k)))*exp(-sqrt(k)*t(101:200));
plot(t,x)
hold on
k=4

k =

     4

x(1:100)=(-1/(2*sqrt(k)))*exp(sqrt(k)*t(1:100));
x(101:200)=(-1/(2*sqrt(k)))*exp(-sqrt(k)*t(101:200));
plot(t,x)
k=10

k =

    10

x(1:100)=(-1/(2*sqrt(k)))*exp(sqrt(k)*t(1:100));
x(101:200)=(-1/(2*sqrt(k)))*exp(-sqrt(k)*t(101:200));
plot(t,x)
diary off
```

5

$$20\log|H(s)| = 20 + 20\log|s| + 20\log|s+100| - 20\log|s+1| - 20\log|s+1000|$$

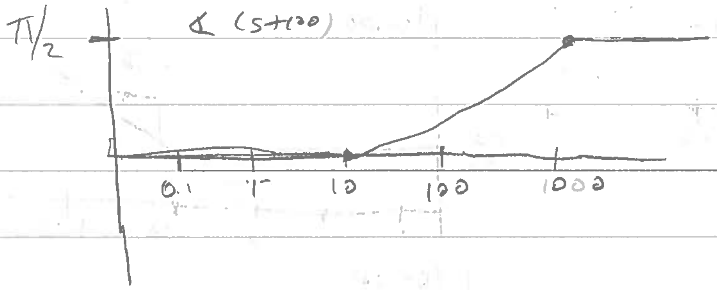


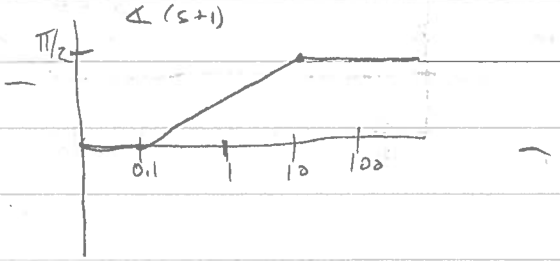



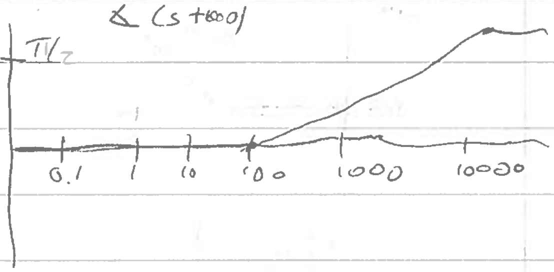

Phase on back) →

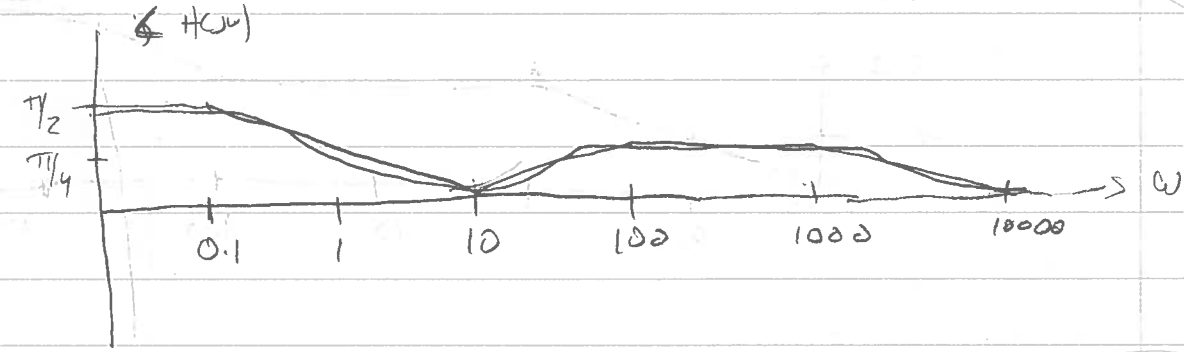
$$\measuredangle H(s) = \measuredangle 10 + \measuredangle s + \measuredangle (s+100) - \measuredangle (s+1) - \measuredangle (s+1000)$$

$$= 0 + \pi/2 + \measuredangle (s+100) - \measuredangle (s+1) - \measuredangle (s+1000)$$

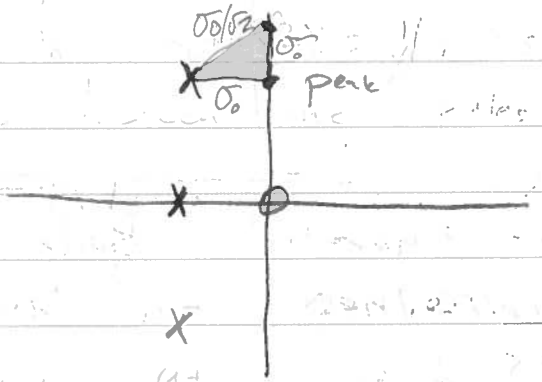


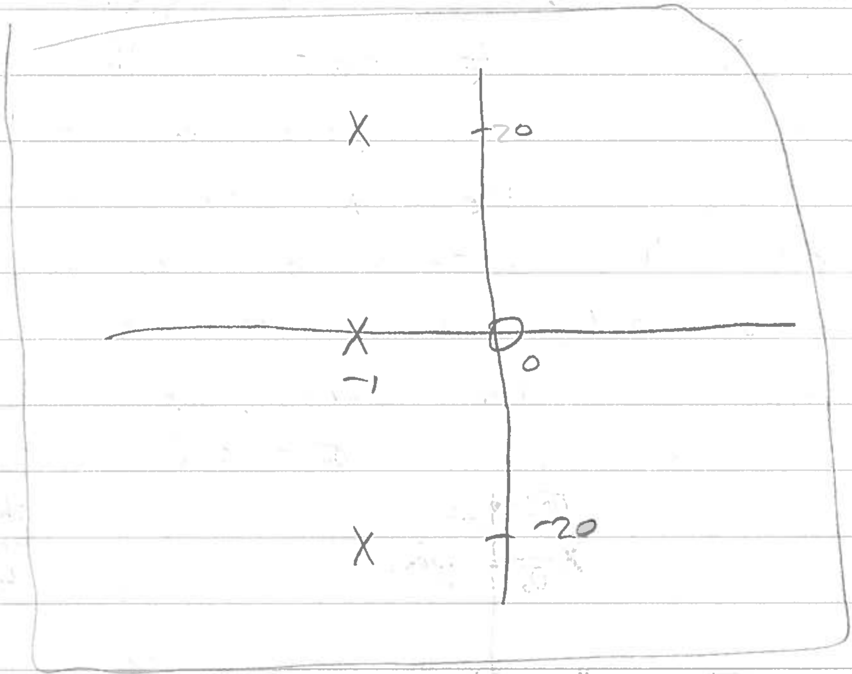



At low frequency we have an upward slope of 20 dB/dec (asymptotically). This is consistent with a zero at zero. We also see on the magnitude plot a breakpoint at $\omega = 1$ at which the slope decreases from 20 dB/dec to zero. This is consistent with a pole at $-1$. Finally, we see a peak at $\omega = 20$. This corresponds to a complex pair of poles with imaginary part $\pm 20$. To find the real part, we suppose that the poles are $-\sigma_0 \pm 20j$. (Inspection of ~~the phase plot~~ shows the poles are left of the $j\omega$ axis.)



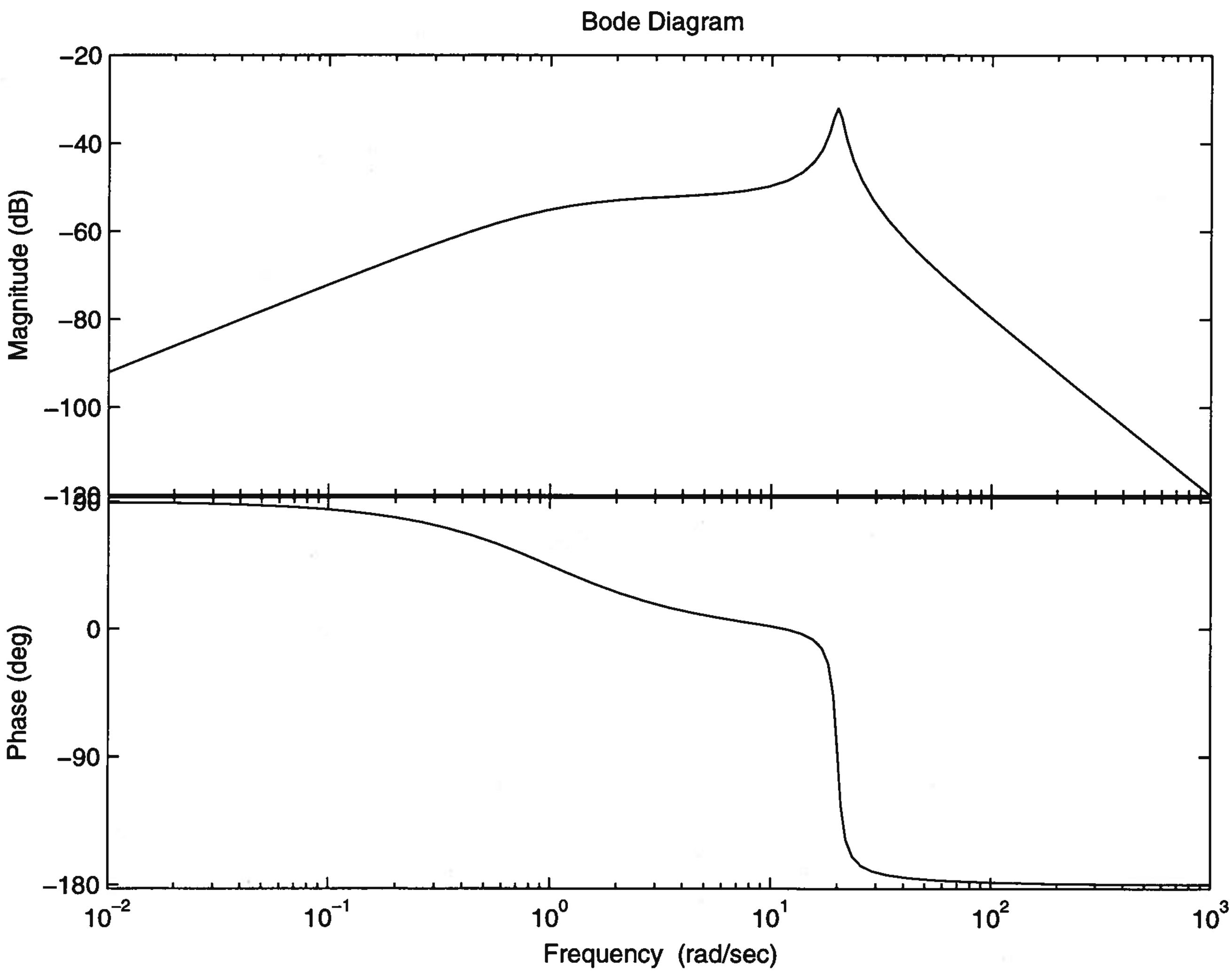

Consider the point $j(20 + \sigma_0)$, which is indicated on the graph. We suppose $\sigma_0 \ll 20$, so the distance to the zero and the poles at 1 and ~~$-\sigma$~~ $-20$ does not change much as we move from the peak $j20$ to $j(20 + \sigma_0)$. However, the distance from the nearest

Thus we expect the amplitude to drop by a factor of $\sqrt{2}$ as we move. So $\sigma_0$ is the distance from the peak to the -3dB points. By inspection of the graph, this is 1. So $\sigma_0 = 1$




A Matlab-generated Bode plot verifies the correctness of this result.

6.


Bode Diagram
Magnitude (dB)
−20
−40
−60
−80
−100
−120
Phase (deg)
90
0
−90
−180
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
Frequency (rad/sec)

```
diary on
s=tf('s')

Transfer function:
s

sys=s/((s+1)*(s+1+20i)*(s+1-20i))
Warning: Transfer function has complex coefficients.
> In /mit/matlab_v6.5/distrib/toolbox/control/control/@tf/tf.m at line 197
  In /mit/matlab_v6.5/distrib/toolbox/control/control/@tf/plus.m at line 26
Warning: Transfer function has complex coefficients.
> In /mit/matlab_v6.5/distrib/toolbox/control/control/@tf/tf.m at line 197
  In /mit/matlab_v6.5/distrib/toolbox/control/control/@tf/plus.m at line 26
  In /mit/matlab_v6.5/distrib/toolbox/control/control/@lti/minus.m at line 12

Transfer function:
            s
-------------------------
s^3 + 3 s^2 + 403 s + 401

bode(sys)
diary off
```

7. a.

$$H(s) = \frac{k\,(s - 2\pi 2600j)(s + 2\pi 2600j)}{(s + D - 2\pi 2600j)(s + D + 2\pi 2600j)}$$

$$H(s) = k\left(\frac{s^2 + (2\pi 2600)^2}{s^2 + 2Ds + D^2 + (2\pi 2600)^2}\right)$$

$$|H(j\omega)| = |k|\,\frac{|(j\omega)^2 + (2\pi 2600)^2|}{|(j\omega)^2 + 2Dj\omega + D^2 + (2\pi 2600)^2|}$$

$$\lim_{\omega \to \infty} |H(j\omega)| = 1 \Rightarrow k = 1$$

$$H(s) = \frac{s^2 + (2\pi 2600)^2}{s^2 - 2Ds + D^2 + 2\pi 2600^2}$$

$$= \boxed{\frac{(s - 2\pi 2600j)(s + 2\pi 2600j)}{(s + D - 2\pi 2600j)(s + D + 2\pi 2600j)}}$$

simpler expression

b. By the eigenfcn property, $x(t) = \cos(2\pi 2600 t)$

$$\Rightarrow y(t) = |H(2\pi 2600j)| \cos(2\pi 2600 t + \angle H(2\pi 2600))$$

$$= \boxed{0} \quad \text{since } 2\pi 2600j \text{ is a zero}$$

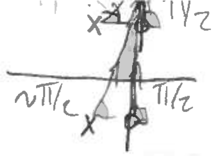
Similarly, if $x(t) = \cos((2\pi 2600 + D)t)$,

we need $|H(j(2\pi 2600 + D))|$ and $\angle H(j(2\pi 2600 + D))$

$$|H(j(2\pi 2600 + D))| = \frac{|Dj| \cdot |j(4\pi 2600 + D)|}{|Dj + D| \cdot |j(4\pi 2600 + D) + D|}$$

$$\approx \frac{D \cdot (4\pi 2600 + D)}{\sqrt{2}D \cdot (4\pi 2600 + D)} \quad (\text{since } 4\pi 2600 \gg D)$$

$$= \frac{1}{\sqrt{2}}$$



(assuming $4\pi 2600 >> D$ for the approximation on the lower left pole)

$$\angle H(j(2\pi 2600+D)) = \pi/2 + \pi/2 - \pi/4 - \pi/2 = \pi/4$$

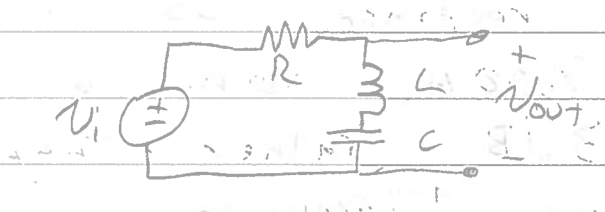
So the response is $\boxed{\frac{1}{\sqrt{2}} \cos\left((2\pi 2600+D)t + \pi/4\right)}$

by the eigenfunction property.

c. We define the bandwidth of the notch filter to be the distance between -3 dB points. We showed above that the notch filter is centered at $\omega = 2\pi 2600$ and at $\omega = 2\pi 2600 + D$ has attenuation $1/\sqrt{2}$ = -3 dB. We can show similarly (by the symmetry of the problem) that there is also -3 dB attenuation at $\omega = 2\pi 2600 - D$. See also the attached Bode plots. We want $D$ to be small so we will filter out the unwanted 2600 Hz tone while retaining as many other frequencies as possible with gain near 1.

d. See the attached Matlab plots of step responses. As $D$ is decreased (narrower bandwidth), the step response takes longer to reach steady-state (note the different axis scales on the plots). Thus there is a trade-off between narrower bandwidth and time to reach steady state.

e. Consider a circuit




Then $V_{out} = \frac{sL + 1/sC}{R + sL + 1/sC} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

We want this to equal

$$\frac{s^2 + (2\pi 2600)^2}{s^2 + 2Ds + D^2 + (2\pi 2600)^2}$$

(from part A.

So $\frac{1}{LC} = (2\pi 2600)^2 \Rightarrow$

$$L = \frac{1}{C(2\pi 2600)^2} = \frac{1}{1\mu F(2\pi 2600)^2}$$

$$= \boxed{3.747 \text{ mH}}$$

and $2Ds = \frac{R}{L}s \Rightarrow R = 2DL = \boxed{1.224\ \Omega}$

f. Suppose the inductor has a 2% tolerance. Then $L_{2\%}$ could be as high as $1.02L$ $= 3.822$ mH. (It could also be as low as $.98L$; the argument is similar in this case). Then the center frequency of the notch will be

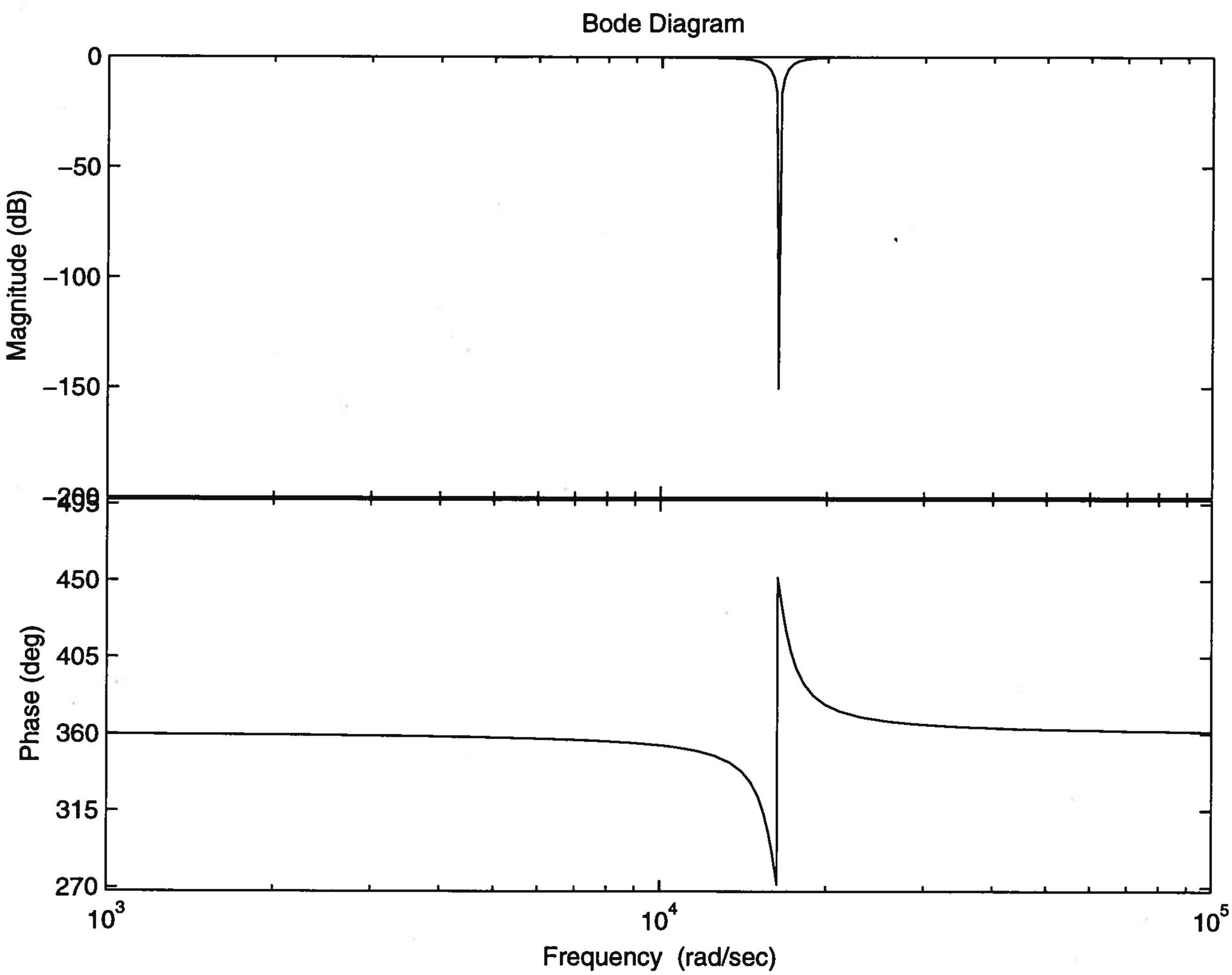
differs from $2\pi \cdot 2600$ Hz by 161 rad/s.
But the bandwidth of the filter is
$2\pi \cdot 26 = 163$. So the desired 2600 Hz
frequency is almost at the 3 dB point
of the frequency response. So it will
attenuate the 2600 Hz tone by
only about 3 dB rather than the
desired total attenuation.

e) We can make the circuit more
tolerant of variance in the inductance
by choosing R, L, C so that the
bandwidth D is larger. The
disadvantage is that we filter
out other frequencies (as in part c.)

7c

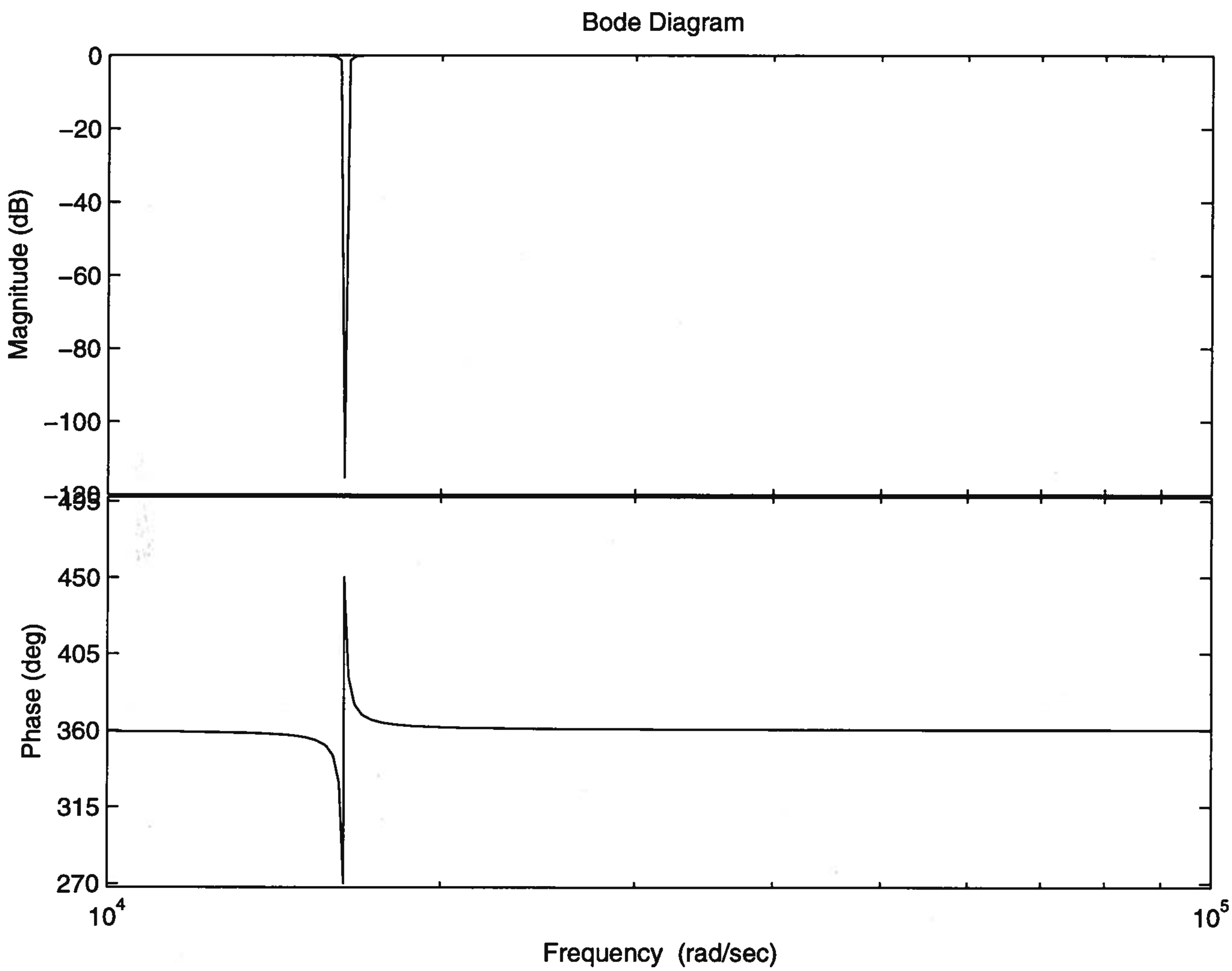
7c

D=1000



Bode Diagram
Magnitude (dB)
0
−50
−100
−150
−200
Phase (deg)
495
450
405
360
315
270
$10^3$
$10^4$
$10^5$
Frequency (rad/sec)

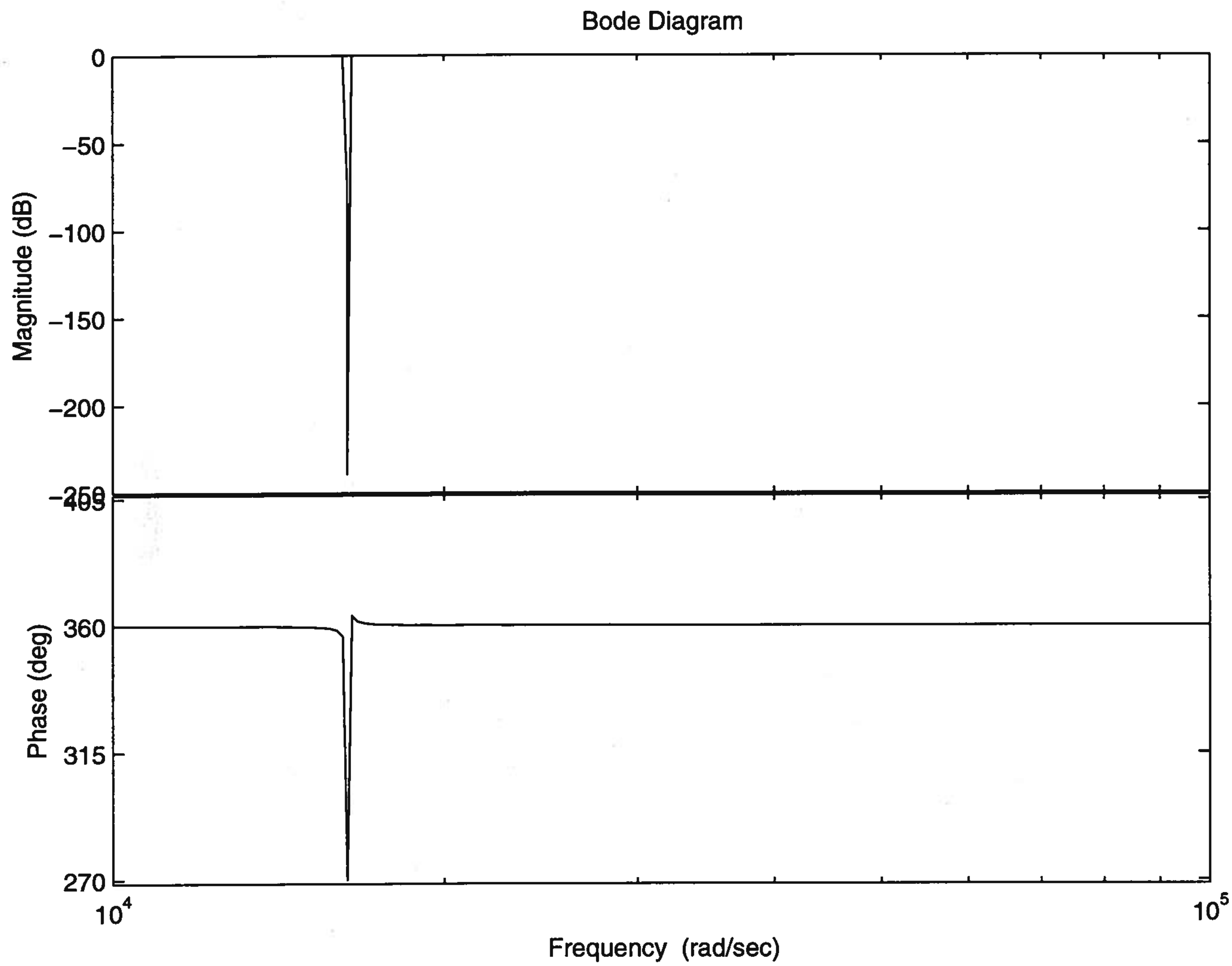
7c.    $D = 100$    7c


Bode Diagram
0
−20
−40
−60
−80
−100
Magnitude (dB)
450
405
360
315
270
Phase (deg)
10^4
10^5
Frequency (rad/sec)

7c.

D = 10

7c



Bode Diagram
Magnitude (dB)
0
−50
−100
−150
−200
−250
405
360
315
270
Phase (deg)
$10^4$
$10^5$
Frequency (rad/sec)

```
diary on
s=tf('s')

Transfer function:
s

D=1000

D =

        1000

sys=(s-2*pi*2600*i)*(s+2*pi*2600*i)/((s+D-2*pi*2600*i)*(s+D+2*pi*2600*i))
Warning: Transfer function has complex coefficients.
> In /mit/matlab_v6.5/distrib/toolbox/control/control/@tf/tf.m at line 197
  In /mit/matlab_v6.5/distrib/toolbox/control/control/@tf/plus.m at line 26
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Transfer function:
    s^2 + 2.669e08
-----------------------
s^2 + 2000 s + 2.679e08

bode(sys)
D=100

D =

   100

sys=(s-2*pi*2600*i)*(s+2*pi*2600*i)/((s+D-2*pi*2600*i)*(s+D+2*pi*2600*i))
Warning: Transfer function has complex coefficients.
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Transfer function:
    s^2 + 2.669e08
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s^2 + 200 s + 2.669e08

bode(sys)
D=10

D =
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sys=(s-2*pi*2600*i)*(s+2*pi*2600*i)/((s+D-2*pi*2600*i)*(s+D+2*pi*2600*i))
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Transfer function:
   s^2 + 2.669e08
---------------------
s^2 + 20 s + 2.669e08

bode(sys)
diary off
```
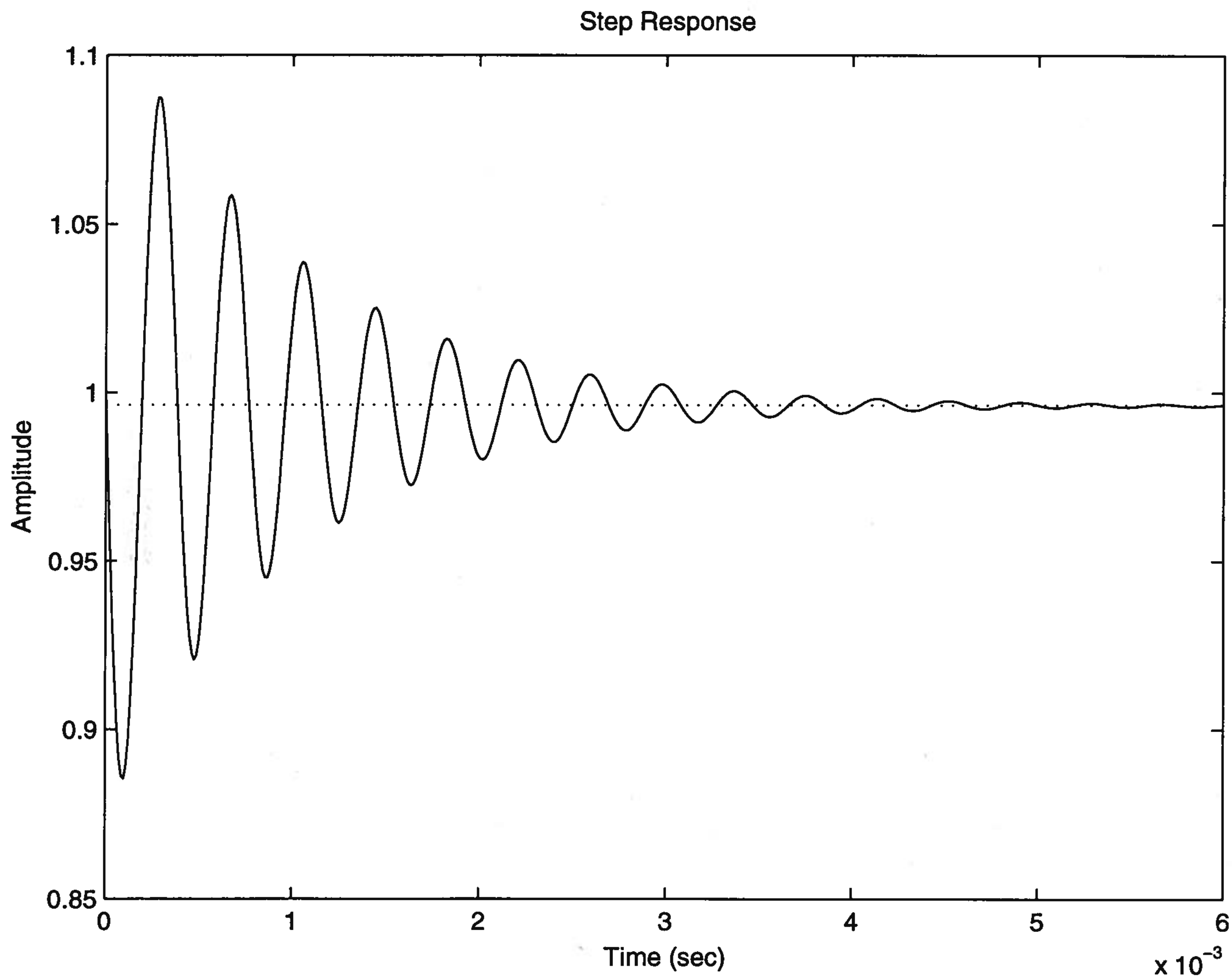

7d

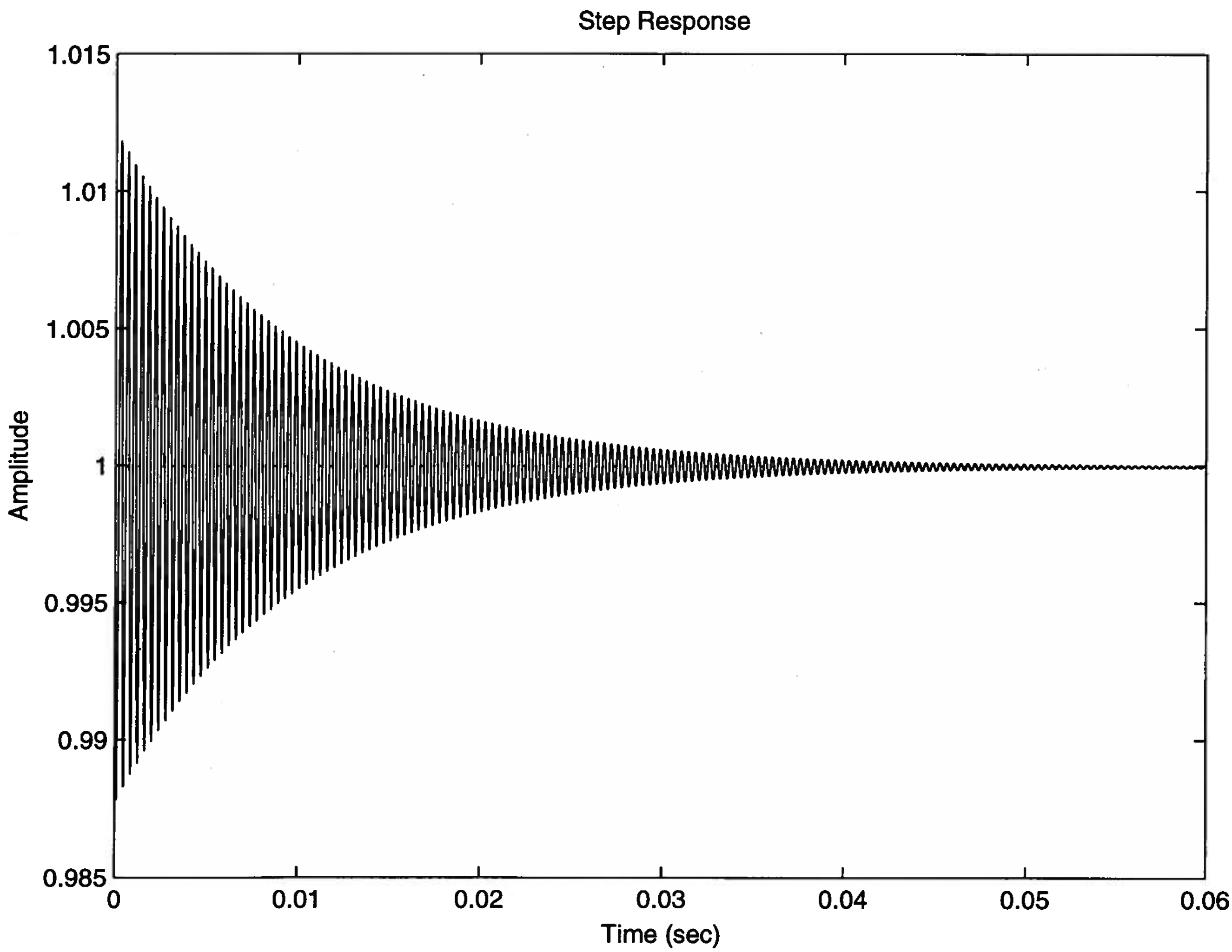
D=1000

7d


Step Response
1.1
1.05
1
0.95
0.9
0.85
Amplitude
0
1
2
3
4
5
6
Time (sec)
x 10^-3

7d.] D=100

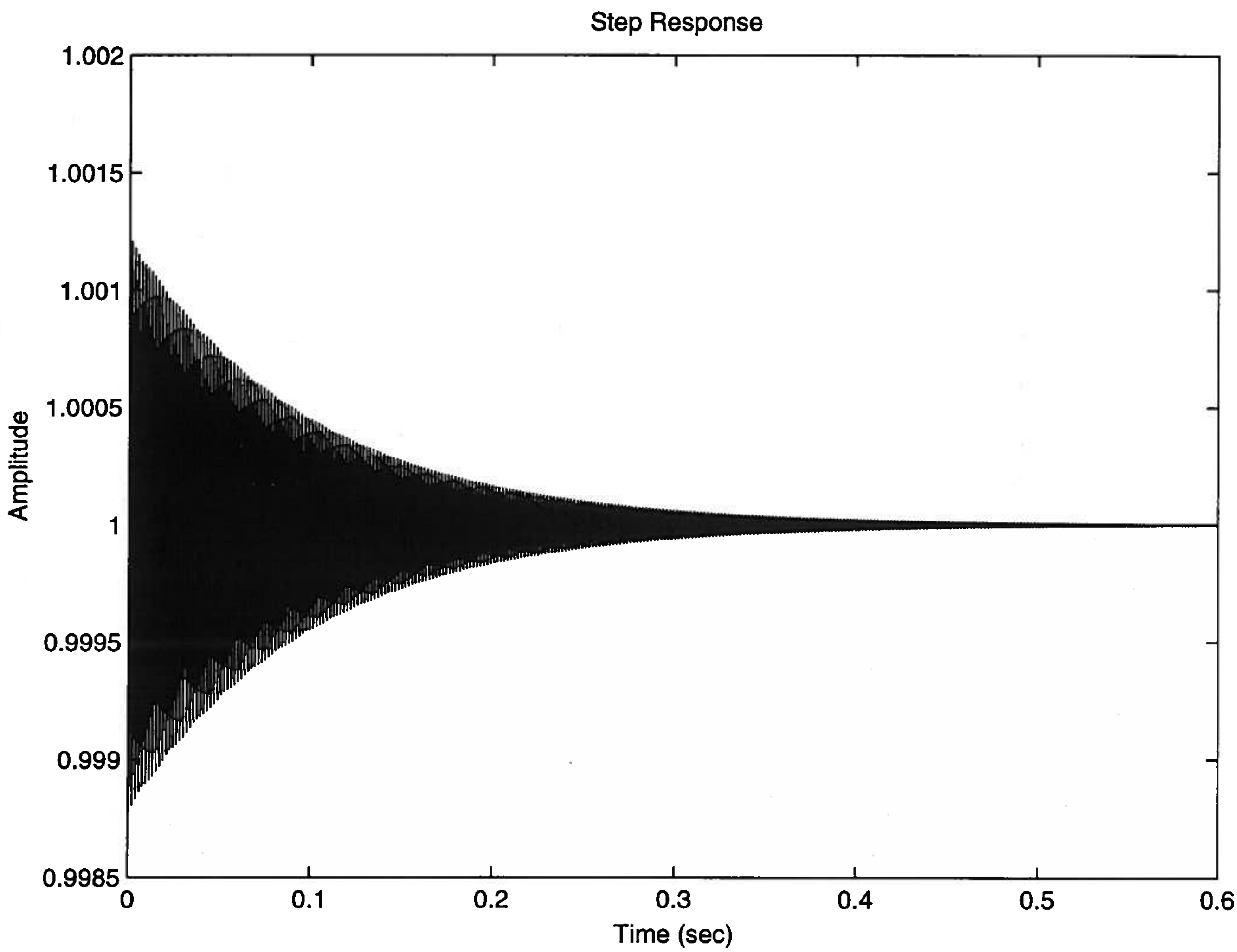
[7d



Step Response
Amplitude
Time (sec)
1.015
1.01
1.005
1
0.995
0.99
0.985
0
0.01
0.02
0.03
0.04
0.05
0.06

7d D=10

7d


Step Response
1.002
1.0015
1.001
1.0005
1
0.9995
0.999
0.9985
Amplitude
0
0.1
0.2
0.3
0.4
0.5
0.6
Time (sec)

```
s=tf('s')

Transfer function:
s

D=1000

D =

        1000

sys=(s-2*pi*2600*i)*(s+2*pi*2600*i)/((s+D-2*pi*2600*i)*(s+D+2*pi*2600*i))
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Transfer function:
    s^2 + 2.669e08
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s^2 + 2000 s + 2.679e08

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   100

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Transfer function:
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Transfer function:
   s^2 + 2.669e08
---------------------
s^2 + 20 s + 2.669e08

step(sys)
diary off
```