Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
Professor Albert Meyer and Dr. Radhika Nagpal

Solutions cover sheet
November 4

Student's Solutions to Problem Set 10

Your name: Dan Portz

Due date: November 8

Submission date: 11/12

Circle your TA: Adrian George Josh Karen Lee Min Nikos Tina

Collaboration statement: Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:

   got help from:[1]

   and referred to:[2]

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	3
2	3
3	3
Total	9

[1]People other than course staff.

[2]Give citations to texts and material other than the class texts, handouts, and last term's bible.

Dan Ports
6.042 PS10

1\. a. Let the probability space be defined by (i, n) where $1 \le i \le 4$ represent the coin that is chosen and $n \ge 0$ be the toss that gives heads. This requires that coin i be chosen with probability $1/4$, that it be flipped $n-1$ times giving tails, then once to give heads. The probability of this is $\boxed{\Pr\{(i,n)\} = \frac{1}{4}\left(1-\frac{1}{i}\right)^{n-1} \cdot \left(\frac{1}{i}\right)}$ ✓

The sum of the probabilities in this space is $\sum_{i=1}^{4} \sum_{n=1}^{\infty} \frac{1}{4}\left(1-\frac{1}{i}\right)^{n-1} \frac{1}{i}$

$$= \frac{1}{4}\sum_{n=1}^{\infty}\left(0^{(n-1)} + \left(\frac{1}{2}\right)^{n-1}\cdot\frac{1}{2} + \left(\frac{2}{3}\right)^{n-1}\cdot\frac{1}{3} + \left(\frac{3}{4}\right)^{n-1}\cdot\frac{1}{4}\right)$$

$$= \frac{1}{4}\left[1 + \frac{1}{1-1/2}\cdot\frac{1}{2} + \frac{1}{1-2/3}\left(\frac{1}{3}\right) + \frac{1}{1-3/4}\left(\frac{1}{4}\right)\right]$$

$$= \frac{1}{4}(1+1+1+1) = 1, \text{ as required.}$$

b. $\Pr\{H_2\} = \sum_{i=1}^{4} \frac{1}{4}\left(1-\frac{1}{i}\right)^{2-1} \cdot \left(\frac{1}{i}\right)$

$$= \frac{1}{4}\left[(1-1)\cdot 1 + (1-1/2)(1/2) + (1-1/3)\cdot 1/3 + (1-1/4)\cdot 1/4\right]$$

$$= \frac{1}{4}\left[0 + \frac{1}{2}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{3} + \frac{3}{4}\cdot\frac{1}{4}\right]$$

$$= \frac{1}{4}\left[\frac{1}{4} + \frac{2}{9} + \frac{3}{16}\right] = \boxed{95/576}$$ ✓

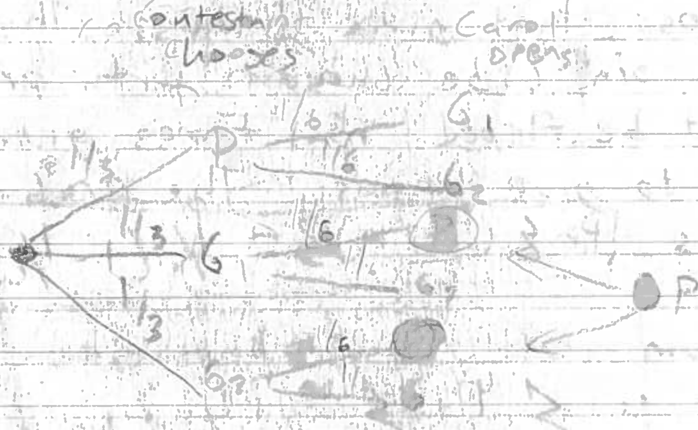
c. $\Pr\{C_i \mid H_2\} = \frac{\Pr\{H_2 \mid C_i\}\Pr\{C_i\}}{\Pr\{H_2\}}$

$$= \frac{(1-1/i)(1/i)\cdot 1/4}{95/576} = \boxed{\left(1/i - 1/i^2\right)\cdot\frac{144}{95}}$$ ✓

d. $\sum_{i=1}^{4} \Pr\{C_i \mid H_2\} = \frac{144}{95} \cdot \left[1/1 - 1/1 + 1/2 - 1/4 + 1/3 - 1/4 + 1/4 - 1/10\right]$

$= \frac{144}{95} \cdot \frac{95}{144} = \boxed{1}$ as expected.

2. a. Let the doors be labeled by what is behind them: goats G_1 and G_2 and prize P.



From the tree $\Pr\{OP\} = 1/6 \cdot 2 = \boxed{1/3}$

Again from the tree

$\Pr\{GP \mid \overline{OP}\} = \frac{1/6 + 1/6}{4/6} = \boxed{1/2}$

b. This requires OP $n-1$ times then $\overline{OP}$

$[\Pr\{OP\}]^{n-1} \Pr\{\overline{OP}\} = \boxed{(1/3)^{n-1} \cdot (1 - 1/3)}$

c. For the game to end at n guesses, we have

$[\Pr\{OP\}]^{n-1} \cdot \Pr\{\overline{OP}\} = (1/3)^{n-1} \cdot (1 - 1/3) = (1/3)^{n-1} - (1/3)^n$

The limit as $n \to \infty$ is 0. So it is not strictly impossible that the game goes on forever, but the probability is zero so we can discount it as a possibility.

d. Consider a space defined by (m, x) where m is the number of choices the player makes before the game ends and x is the outcome: win or lose. W_n consists of the outcomes (y, WIN) where $y \geq n$.