Massachusetts Institute of Technology
6.042J/18.062J, Fall '02: Mathematics for Computer Science
Professor Albert Meyer and Dr. Radhika Nagpal

Solutions cover sheet
October 28

Student's Solutions to Problem Set 9

Your name: Dan Ports

Due date: November 4

Submission date: 11/4

Circle your TA: Adrian George Josh Karen Lee Min Nikos Tina

Collaboration statement: Circle one of the two choices and provide all pertinent info.

1. I worked alone and only with course materials.
2. I collaborated on this assignment with:

   got help from:[1]

   and referred to:[2]

DO NOT WRITE BELOW THIS LINE

Problem	Score
1	3
2	3
3	3
4	3
5	3
6	3
Total	18

[1] People other than course staff.

[2] Give citations to texts and material other than the class texts, handouts, and last term's bible.

Dan Porter
6.042 · PS4

9.1. The committee can have either 4, 5, or 6 Republicans, so

$$\binom{15}{4}\binom{10}{2} + \binom{15}{5}\binom{10}{1} + \binom{15}{6}\binom{10}{0} = \boxed{96460}$$

9.2. One of the digits is a 9; there are 6 choices for which one. The other five add to 4. Consider this as a stars and bars problem where each star represents one unit and each bar separates two digits. There are four stars that can be placed between four bars, so $6 \cdot \binom{8}{4} = \boxed{420}$

9.3. a) From problem 3.5b, we know that for every equivalence relation E there exists a function $f(a) = [a]_E$ such that $a_1 E a_2 \Leftrightarrow f(a_1) = f(a_2)$, where $[a]_E$ is the representative element of the equivalence class containing a. Without loss of generality, assume that the n elements are arbitrarily numbered $a_1, a_2 \ldots a_n$, and that the representative element of an equivalence class is the element of that equivalence class with the lowest index.

We know that $f(a_1)$ then necessarily equals a_1, because a_1 will be the minimal element of any equivalence class that contains a_1. Similarly, $f(a_2)$ can equal a_1 if $a_1 E a_2$ or a_2 otherwise. For some element a_k, there are at most k options for $f(a_k)$. Therefore an upper bound on the number of possible functions $f(a)$ and thus on the number of equivalence relations is $\prod_{k=1}^{n} k = n!$

✓

b. Consider the equivalence class containing some arbitrary element a_1. Without considering any other equivalence classes, we know for any element $a \neq a_1$, either $a E a_1$ is true or it is false. These possibilities are independent, so there are 2^{n-1} possible ways to choose the elements in the equivalence class of a_1. This means there must be at least 2^{n-1} possible equivalence relations with n elements.

✓

4. a. $\binom{2n}{2} = \frac{2n!}{2!(2n-2)!} = \frac{2n(2n-1)}{2}$

$= n(2n-1) = 2n^2 - n = n^2 - n + n^2$

✓ $= n(n-1) + n^2 = \frac{2n!}{2!(n-2)!} + n^2 = 2\binom{n}{2} + n^2$

b. Consider the problem of choosing two elements from a set of $2n$. There are $\binom{2n}{2}$ possible combinations. The set can also be divided into two sets of n elements. To choose two from these, we can either choose two from the first set or two from the second set, with $\binom{n}{2}$ possibilities each, or choose one from the first and one from the second, with $n \cdot n$ possibilities. ✓ By the addition rule, this gives $2\binom{n}{2} + n^2$, so $2\binom{n}{2} + n^2$ must equal $\binom{2n}{2}$

5. Assume the tables are distinct, and choose 4 groups of 3. Then divide by the number of permutations of the tables:

$$\frac{\binom{12}{3}\binom{9}{3}\binom{6}{3}\binom{3}{3}}{4!} = \boxed{15400}$$ ✓

6. a. Choose any four of the 52 positions without regard for ordering.

$$\boxed{\binom{52}{4} = 270725.}$$ ✓

b. Consider this as a stars and bars problem with kings as bars and non-kings as stars. We know that three stars must be placed as |*|*|*| to keep the bars from being adjacent, so ignore these three. The other 45 stars can be placed in

$$\binom{45+4}{4} = \boxed{\binom{49}{4} = 211876}$$ ways. ✓

c. Same as above except that the ordering of the 4 kings and 48 nonkings matters:

$$\boxed{\binom{49}{4} \cdot 4! \cdot 48!}$$ ✓