

```

#####
;;; 6.891 PS2
;;;
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;;;
;;; 2007/02/21
;;;
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;;;
#####

```

```

(load "ghelper.scm")
(load "generic-sequences.scm")

;;;
;;; Problem 1
;;;
;;; Implementations of the sequence operators are below.
;;;
;;; One notable implementation decision: many of the functions, such
;;; as map, are implemented in terms of streams. Many functions
;;; need to iterate over sequences in order, which needs to be done
;;; differently in order to be efficient for each data type: for
;;; example, we can reference each arbitrary index in a vector with
;;; sequence.ref, but for a list it's much faster to cdr down the
;;; list. So we define a sequence:stream operation that generates a
;;; stream from a sequence. The default implementation uses
;;; sequence.ref to fetch each element, but it's specialized for
;;; lists. Since it's a stream, we don't need to access elements until
;;; they become necessary (which makes this more efficient than
;;; coercing other sequence types into lists). It also makes it
;;; possible to mix different types of sequences as arguments without
;;; coercing (but that's Problem 2).

```

```

;;; Some helper functions
(define (assert x . msg)
  (if (not x) (apply error msg)))

(define (assert-equal a b)
  (assert (equal? a b)
    "Test failed"
    "Expected: "
    b
    " got: "
    a)
  #t)

(define (list-of? pred?)
  (lambda (lst)
    (and
     (list? lst)
     (for-all? list pred?))))

(assert-equal ((list-of? string?) "a") #f)
(assert-equal ((list-of? string?) ("a" "b")) #t)
(assert-equal ((list-of? string?) ("a" 3)) #f)

;;; CONSTRUCT
(define generic:construct
  (make-generic-operator 2 #f))

(assign-operation generic:construct

```

```

(lambda (type items)
  (is-exactly list?) any?)
(assign-operation generic:construct
  (lambda (type items) (list->vector items))
  (is-exactly vector?) any?)
(assign-operation generic:construct
  (lambda (type items) (list->string items))
  (is-exactly string?) any?)
(define (sequence:construct type . items)
  (generic:construct type items))

```

```

(assert-equal (sequence:construct list? 0 1 2 3 4) '(0 1 2 3 4))
(assert-equal (sequence:construct vector? 0 1 2 3 4) #(0 1 2 3 4))
(assert-equal (sequence:construct string? #\a #\b #\c) "abc")

;;; Generate
(define sequence:generate
  (make-generic-operator 3 #f))
(define (range start end)
  (if (>= start end)
      '()
      (cons start (range (+ 1 start) end))))
(define (list-generate n fn)
  (map fn (range 0 n)))
(define (string-generate n fn)
  (apply string-append
    (list-generate n fn)))

```

```

(assign-operation sequence:generate
  (lambda (type n fn) (string-generate n fn))
  (is-exactly string?) exact-integer? any?)
(assign-operation sequence:generate
  (lambda (type n fn) (list-generate n fn))
  (is-exactly list?) exact-integer? any?)
(assign-operation sequence:generate
  (lambda (type n fn) (make-initialized-vector n fn))
  (is-exactly vector?) exact-integer? any?)

(assert-equal
  (sequence:generate list? 5 (lambda (x) x))
  '(0 1 2 3 4))
(assert-equal
  (sequence:generate string? 5 (lambda (x) (string (digit->char x))))
  "01234")
(assert-equal
  (sequence:generate vector? 5 (lambda (x) x))
  #(0 1 2 3 4))

;;; stream (convert sequence to stream)
(define (generic-sequence-stream sequence)
  (let loop ((i 0))
    (if (= i len)
        '()
        (cons-stream (sequence:ref sequence i)
                      (loop (+ i 1))))))

```

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ps2.scm

WELLDONE!

LIST, ... BUT OK BY CONSTRUCTION

iota [SEE MIT SCHEME REF MAN]

make-initialized-list

OTHER FOLKS RESTRICTED FN TO RETURN A CHAR, SO THEY JUST USED list->string.

procedure?

empty-stream

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NICE

NICE!!

NICE

```

(define sequence:stream
  (make-generic-operator 1 generic-sequence-stream))

(assign-operation sequence:stream
  (list->stream
   list?))

(assert-equal
 (stream->list
  (sequence:stream (sequence:generate vector? 5 (lambda (x) x))))
 (sequence:generate list? 5 (lambda (x) x)))

(assert-equal
 (stream->list
  (sequence:stream (sequence:generate list? 5 (lambda (x) x))))
 (sequence:generate list? 5 (lambda (x) x)))

(assert-equal
 (stream->list
  (sequence:stream (sequence:generate string? 5 (lambda (x) (string (digit->char x))))))
 (sequence:generate list? 5 (lambda (x) (digit->char x))))

;; map

(define (generic-sequence-map type fn sequences)
  (apply sequence:construct
   type
   (let loop ((streams (map sequence:stream sequences)))
     (if (stream-null? (car streams))
         '()
         (let ((val (apply fn (map stream-car streams))))
           (cons val
                  (loop (map stream-cdr streams))))))))

(define generic:map
  (make-generic-operator 3 generic-sequence-map))

(assign-operation generic:map
  (lambda (type fn l) (apply map (cons fn l)))
  (is-exactly list?) any? (list-of? list?))

(define (sequence:map fn . sequences)
  ;; XXX Should assert that sequences are of the same type, but see
  ;; Prob. 2
  (generic:map (sequence:type (car sequences))
               fn
               sequences))

(assert-equal
 (sequence:map (lambda (x) (+ 1 x))
               (sequence:generate list? 5 (lambda (x) x)))
 '(1 2 3 4 5))

(assert-equal
 (sequence:map (lambda (x) (+ 1 x))
               (sequence:generate vector? 5 (lambda (x) x)))
 #(1 2 3 4 5))

(assert-equal
 (sequence:map char-upcase "abc")
 "ABC")

(assert-equal
 (sequence:map * '(1 2 3 4 5) '(10 20 30 40 50))
 '(10 40 90 160 250))

;; for-each

```

```

(define (generic-sequence-for-each type fn sequences)
  (let loop ((streams (map sequence:stream sequences)))
    (if (stream-null? (car streams))
        #t OR 'DONE
        (let ((val (apply fn (map stream-car streams))))
          (cons val
                 (loop (map stream-cdr streams)))))))

(define generic:for-each
  (make-generic-operator 3 generic-sequence-for-each))

(assign-operation generic:for-each
  (lambda (type fn l) (apply for-each (cons fn l)))
  (is-exactly list?) any? (list-of? list?))

(define (sequence:for-each fn . sequences)
  ;; XXX Should assert that sequences are of the same type, but see
  ;; Prob. 2
  (generic:for-each (sequence:type (car sequences))
                    fn
                    sequences))

(assert-equal
 (let ((lst '()))
   (sequence:for-each (lambda (x) (set! lst (cons x lst)))
                      '(1 2 3 4 5))
   lst)
 '(5 4 3 2 1))

(assert-equal
 (let ((lst '()))
   (sequence:for-each (lambda (x) (set! lstv (cons x lst)))
                      '(1 2 3 4 5))
   lst)
 '(5 4 3 2 1))

(assert-equal
 (let ((lst '()))
   (sequence:for-each (lambda (x) (set! lst (cons x lst)))
                      "abc")
   lst)
 '(#\c #\b #\a))

(assert-equal
 (let ((lst '()))
   (sequence:for-each (lambda (x y) (set! lst (cons (+ x y) lst)))
                      '(1 2 3 4 5) '(10 20 30 40 50))
   lst)
 '(55 44 33 22 11))

;; filter

(define (generic-sequence-filter seq pred)
  (apply sequence:construct
   (sequence:type seq)
   (let loop ((stream (sequence:stream seq))
              (if (stream-null? stream)
                  '()
                  (if (pred (stream-car stream))
                      (cons (stream-car stream)
                            (loop (stream-cdr stream)))
                      (loop (stream-cdr stream))))))

(define sequence:filter
  (make-generic-operator 2 generic-sequence-filter))

```

OK, BUT FOR-EACH NEEDN'T ACCUMULATE RESULT VALS.

ELEGANT!

T-procedure? OK

OK

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```
(assign-operation sequence:filter
  (lambda (seq pred) (list-transform-positive seq pred)
    list? any?)
  procedure?)

(assert-equal
 (sequence:filter
  (sequence:generate list? 6 (lambda (x) x)) odd?)
 '(1 3 5))
(assert-equal
 (sequence:filter
  (sequence:generate vector? 6 (lambda (x) x)) odd?)
 '(1 3 5))
(assert-equal
 (sequence:filter
  (sequence:generate string? 5 (lambda (x) (string (digit->char x))))
  (lambda (x) (char? x #\3)))
 "012")
;; get-index
(define (generic-sequence-get-index seq pred)
  (let loop ((stream (sequence:stream seq))
             (i 0))
    (if (stream-null? stream)
        #f
        (if (pred (stream-car stream))
            i
            (loop (stream-cdr stream) (+ i 1))))))
(define sequence:get-index
  (make-generic-operator 2 generic-sequence-get-index))
(assert-equal
 (sequence:get-index
  (sequence:generate list? 6 (lambda (x) x)) odd?)
 1)
(assert-equal
 (sequence:get-index
  (sequence:generate list? 6 (lambda (x) x)) (lambda (x) (> x 10)))
 #f)
(assert-equal
 (sequence:get-index
  (sequence:generate vector? 6 (lambda (x) x)) odd?)
 1)
(assert-equal
 (sequence:get-index
  (sequence:generate string? 5 (lambda (x) (string (digit->char x))))
  (lambda (x) (char? x #\3)))
 4)
;; get-element
(define (generic-sequence-get-element seq pred)
  (let loop ((stream (sequence:stream seq))
             (if (stream-null? stream)
                 #f
                 (if (pred (stream-car stream))
                     (stream-car stream)
                     (loop (stream-cdr stream))))))
  (define sequence:get-element
    (make-generic-operator 2 generic-sequence-get-element)))

```

For Lists, could use
find-matching-item

```
(make-generic-operator 2 generic-sequence-get-element))
(assert-equal
 (sequence:get-element
  (sequence:generate list? 6 (lambda (x) x)) even?)
 0)
(assert-equal
 (sequence:get-element
  (sequence:generate list? 6 (lambda (x) x)) (lambda (x) (> x 10)))
 #f)
(assert-equal
 (sequence:get-element
  (sequence:generate vector? 6 (lambda (x) x)) odd?)
 1)
(assert-equal
 (sequence:get-element
  (sequence:generate string? 5 (lambda (x) (string (digit->char x))))
  (lambda (x) (char? x #\3)))
 #\4)
;; fold-right
(define (generic-sequence-fold-right f init seq)
  (let loop ((stream (sequence:stream seq))
             (if (stream-null? stream)
                 init
                 (loop (stream-cdr stream))))))
(define sequence:fold-right
  (make-generic-operator 3 generic-sequence-fold-right))
(assert-equal
 (sequence:fold-right + 0 '(1 2 3 4 5))
 15)
(assert-equal
 (sequence:fold-right (lambda (x r) (append r (list x))) '() '(1 2 3 4 5))
 '(5 4 3 2 1))
(assert-equal
 (sequence:fold-right + 0 #'(1 2 3 4 5))
 15)
(assert-equal
 (sequence:fold-right (lambda (x r) (append r (list x))) '() #'(1 2 3 4 5))
 '(5 4 3 2 1))
(assert-equal
 (sequence:fold-right (lambda (x r) (append r (list x))) '() "abcd"))
 '(#\d #\c #\b #\a))
;; fold-left
(define (generic-sequence-fold-left f init seq)
  (let loop ((stream (sequence:stream seq))
             (result init))
    (if (stream-null? stream)
        result
        (loop (stream-cdr stream)
              (f result (stream-car stream)))))
  (define sequence:fold-left
    (make-generic-operator 3 generic-sequence-fold-left)))
(assert-equal
 (sequence:fold-left + 0 '(1 2 3 4 5))
 15)

```

For Lists, could use
fold-right

For Lists, could use
fold-left

```

15)
(assert-equal
 (sequence:fold-left (lambda (x y) (cons y x)) '() '(1 2 3 4 5))
 '(5 4 3 2 1))
(assert-equal
 (sequence:fold-left + 0 #(1 2 3 4 5))
 15)
(assert-equal
 (sequence:fold-left (lambda (x y) (cons y x)) '() #(1 2 3 4 5))
 '(5 4 3 2 1))
(assert-equal
 (sequence:fold-left (lambda (x y) (cons y x)) '() "abcd")
 '#\d #\c #\b #\a))

```

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WELL
DONE

```

;;;
;;; Problem 2
;;;
;;; Several of the sequence functions take multiple sequences. We
;;; remove the restriction that the sequences need to be the same
;;; type:
;;;
;;; (sequence:append <sequence-1> ... <sequence-n>)
;;; Returns a new sequence of the same type as sequence-1, formed
;;; by concatenating the elements of the given sequences. The size
;;; of the new sequence is the sum of the sizes of the given
;;; sequences. Sequences need not be of the same type.
;;;
;;; (sequence:map <function> <seq-1> ... <seq-n>)
;;; Requires that the sequences given are of the same size, and
;;; that the arity of the function is n. The ith element of the
;;; new sequence is the value of the function applied to the n ith
;;; elements of the given sequences. Sequences need not be of the
;;; same type.
;;;
;;; (sequence:for-each <procedure> <seq-1> ... <seq-n>)
;;; Requires that the sequences given are of the same size, and
;;; that the arity of the procedure is n. Applies the procedure to
;;; the n ith elements of the given sequences; discards the value.
;;; This is done for effect. Sequences need not be of the same
;;; type.

```

```

;;; Conveniently, implementing sequence:map and sequence:for-each does
;;; not require any modifications because of our stream-based
;;; implementation. We replace the implementation of sequence:append
;;; with one based on streams.

```

```

(define (generic-sequence-append type sequences)
  (apply sequence:construct
    type
    (apply append
      (map stream->list
        (map sequence:stream sequences))))))

```

append-map!

```

(define generic:append
  (make-generic-operator 2 generic-sequence-append))
(define (sequence:append . sequences)
  (generic:append (sequence:type (car sequences))
    sequences))

```

```

(assert-equal (sequence:append '(1 2 3) '(4 5) '(6) '())
 '(1 2 3 4 5 6))
(assert-equal (sequence:append #(1 2 3) #(4 5) #(6) '())
 #(1 2 3 4 5 6))
(assert-equal (sequence:append "abc" "de" "f" "")
 "abcdef")
(assert-equal (sequence:append '(1 2 3) #(4 5) '(6) '())
 '(1 2 3 4 5 6))
(assert-equal (sequence:append #(1 2 3) '(4 5) '(6) '())
 #(1 2 3 4 5 6))

```

```

(assert-equal
 (sequence:map * '(1 2 3 4 5) #(10 20 30 40 50))
 '(10 40 90 160 250))
(assert-equal
 (sequence:map * #(1 2 3 4 5) '(10 20 30 40 50))
 #(10 40 90 160 250))

```

Good

ps2.scm

```
(assert-equal
 (let ((lst '()))
   (sequence:for-each (lambda (x y) (set! lst (cons (+ x y) lst)))
     '(1 2 3 4 5) #(10 20 30 40 50))
   lst)
 '(55 44 33 22 11))
```

```
;;;
;;; Problem 3
;;;
```

```
;;; Many of the procedures above (sequence:map, sequence:for-each,
;;; sequence:append, ...) have variable arity, and their
;;; implementations take the following form: a (non-generic) wrapper
;;; function takes its arguments, turns most of them into a list, and
;;; calls a fixed-arity generic procedure. This is a somewhat awkward
;;; dance, and the common pattern suggests that a bit of syntactic
;;; sugar to build it might be useful.
```

```
;;; We could implement a function for building such wrappers, or more
;;; generally, add support for variable-arity functions into the
;;; generic dispatch system by adding support for predicates over the
;;; *entire* argument list, not separate predicates for each
;;; individual argument. The downside is that running more complicated
;;; predicates over the full argument list could be less efficient.
```

```
;;; To implement this, we'd need to modify assign-operation to take a
;;; single full-argument-list predicate rather than a list of
;;; per-argument predicates, and modify the operator function to test
;;; a single predicate against the full argument list rather than
;;; individual predicates against each argument.
```

Good.

2/12

```

;; Compare two elements according to type. Return an error if they
;; have the same type. (This condition is included for safety, as it
;; ensures that generic:less? can't be used if a predicate for
;; comparing two elements of a particular type isn't added. It could
;; be removed so that generic:less? would always return #f instead in
;; this case.)
(define (types? a b)
  (let ((a-ind (sequence:get-index type-ordering
                                   (lambda (type-pred) (type-pred a))))
        (b-ind (sequence:get-index type-ordering
                                   (lambda (type-pred) (type-pred b))))
        (not-a-ind (error "Unknown type for" a))
        (not-b-ind (error "Unknown type for" b))
        (eqv? a-ind b-ind)
        (error "Attempting to compare elements of the same type" a b))
    (cond (eqv? a-ind b-ind)
          (else (< a-ind b-ind))))
)

```

```

(define generic:less?
  (make-generic-operator 2 type=?))
(assign-operation generic:less? null? null?)
(assign-operation generic:less? boolean? boolean?)
(assign-operation generic:less? char? char?)
(assign-operation generic:less? < number? number?)
(assign-operation generic:less? symbol? symbol?)
(assign-operation generic:less? string? string?)
(assign-operation generic:less? vector? vector?)
(assign-operation generic:less? list? list?)

```

```

(assert-equal (generic:less? '() '()) #f)
(assert-equal (generic:less? #f #f) #f)
(assert-equal (generic:less? #t #f) #f)
(assert-equal (generic:less? #f #t) #t)
(assert-equal (generic:less? #a #b) #t)
(assert-equal (generic:less? #a #a) #f)
(assert-equal (generic:less? #b #a) #f)
(assert-equal (generic:less? 1 2) #t)
(assert-equal (generic:less? 2 1) #f)
(assert-equal (generic:less? 1 1) #f)
(assert-equal (generic:less? 'a 'b) #t)
(assert-equal (generic:less? 'b 'a) #f)
(assert-equal (generic:less? 'a 'a) #f)
(assert-equal (generic:less? "aa" "ab") #t)
(assert-equal (generic:less? "aa" "aa") #f)
(assert-equal (generic:less? "ab" "aa") #f)
(assert-equal (generic:less? #1 2) #2)
(assert-equal (generic:less? #1 2) #1)
(assert-equal (generic:less? #2 1) #1)
(assert-equal (generic:less? #2 1) #2)
(assert-equal (generic:less? #1 2) #1)
(assert-equal (generic:less? #1 2) #2)
(assert-equal (generic:less? #1 2) #1)
(assert-equal (generic:less? #1 2) #2)
(assert-equal (generic:less? #1 2) #1)
(assert-equal (generic:less? #1 2) #2)

```

NICK

```

;; Problem 4
;; a. This code is missing the test that returns false if it
;; encounters an element in list-2 that is greater than than its
;; corresponding element in list-1. As a result, it returns true if
;; any element in list-1 is less than its corresponding element in
;; list-2. Besides not matching the intuitive idea of how lists
;; should be compared, it fails to define a total ordering: both
;; (list? '(1 2) '(2 1)) and (list? '(2 1) '(1 2)) will return
;; true. Chaos will ensue.
;; b. Alyssa's explicit case analysis opts for defining the ordering
;; in a single procedure rather than in n^2 generic dispatch
;; entries. This seems worse for extensibility since adding a new
;; type will require modifying the code for this procedure, rather
;; than using the dispatch system to add new entries. In fact,
;; however, the dispatch system isn't much better: adding a new type
;; requires a dispatch entry for comparing it with any other type in
;; the system (the n^2 problem). To achieve extensibility, what we
;; really want to avoid is the need for implementors of a type to
;; know about all other types in the system, and neither approach
;; avoids this problem.

```

EXCELLENT

An alternate approach (and, indeed, the one used below) is to define the total ordering of types in one place. Then a generic:less operator can have a default comparator that compares the types using the total ordering, and use dispatch entries to compare elements of the same type. This avoids the n^2 problem because all an implementor of a new type needs to do is add the type to the total ordering and provide a single generic:less for comparing two elements of that type.

(We define the total ordering in a static list, but it's easy to imagine having functions to modify the list to dynamically add new types into it.) EXACTLY.

```

;; c. An implementation using the above technique follows:
(define (null? a b) #f)
(define (boolean? char < number < symbol < string < vector < list)
  (list null? boolean? char? number? symbol? string? vector? list?))
(define (null? a b) #f)
(define (boolean? a b) (and (not a) b))
;; Alyssa's implementation, lifted straight from the text
(define (list? list-1 list-2)
  (let ((len-1 (length list-1))
        (len-2 (length list-2)))
    (cond ((< len-1 len-2) #t)
          (> len-1 len-2) #f)
    ; Invariant: equal lengths
    (else
     (let prefix? ((list-1 list-1)
                   (list-2 list-2))
       (cond ((null? list-1) #f) ; same
             ((generic:less? (car list-1) (car list-2)) #t)
             ((generic:less? (car list-2) (car list-1)) #f)
             (else (prefix? (cdr list-1) (cdr list-2)))))))
)
(define (vector? a b)
  (list? (vector->list a) (vector->list b)))

```


ps2.scm

```
(generic:sequence->set '(3 2 1)) #f
;;; d) If we hadn't followed Alyssa's suggestion that sets be
;;; represented as sorted, non-redundant lists, and instead had
;;; allowed them to be any sequence (possibly unsorted or redundant),
;;; generic:sequence->set would have been completely trivial, but
;;; would have pushed all of the work into the various set
;;; operations. These operations would either have to sort the
;;; sequence and remove duplicates themselves, giving a  $O(n \lg n)$ 
;;; runtime and silly implementation complexity, or would need  $O(n^2)$ 
;;; algorithms that involved repeatedly iterating over one list to
;;; check which items are in it. In contrast, our set operations can
;;; have linear cost, so we have both improved code simplicity and
;;; faster runtime.
```

EXACTLY!

```
;;; Problem 5
;;;
;;; The choice between dispatching on predicates and dispatching on
;;; tags is a tradeoff between flexibility and speed. To dispatch
;;; based on predicates requires testing the arguments against the
;;; predicates until a match is found. This can be difficult, since
;;; there may be many predicates to test against. More significantly,
;;; however, the tests are arbitrary code, requiring at least the
;;; overhead of a function call and at worst never terminating. It's
;;; not unreasonable that a predicate would need to examine the
;;; contents of its arguments (such as testing whether each element in
;;; a list is a character), which is a linear-time operation, and even
;;; more complex predicates aren't inconceivable. Indeed, if we
;;; support variable-arity operators, complex predicates can become
;;; common.
;;;
;;; To dispatch based on tags, we simply need to examine the tags of
;;; each argument and compare them to entries in a dispatch
;;; table. This requires only a single equality check against the tags
;;; of possible handlers. Using a hash table or other data structures,
;;; it's possible to reduce the number of comparisons that need to be
;;; done in the average case --- but the same is true of predicate
;;; dispatch, since we may be able to structure the data to limit the
;;; number of predicates that need to be tested on average. For
;;; example, a tree with the most common predicates near the top means
;;; that the less common predicates do not need to be tested unless
;;; they are necessary. But predicate testing remains more expensive
;;; than tag checking.
;;;
;;; Predicate dispatch does provide more general operations, since we
;;; can specify predicates more complex than types. For example, we
;;; can define an operator that works one way for positive numbers and
;;; a different way for negative numbers. We couldn't do this with
;;; tagged data unless we had a separate tag for positive and negative
;;; numbers, which would mean we'd have to anticipate the need to
;;; handle positive and negative numbers differently at the time we
;;; created the data, rather than at the time we created each
;;; operation for the operator. (Also, this sort of tagging would
;;; quickly grow hopelessly cumbersome as we add more predicates ---
;;; we'd have to tag numbers as being positive-odd-nonprime numbers,
;;; etc.)
```

2/12

WELL DONE

WELL ARGUED. SEE ALSO THE
RECOMMENDED READING BY M. ERNST.