MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

Physics 8.02 Spring 2002

EXAM 3

Monday, May 6, 2002

P	O	R	T	S											

FAMILY (Last) NAME

D	A	N	I	E	L										

GIVEN (First) NAME

9	6	8	8	1	9	9	2	7

STUDENT ID NUMBER

Your Recitation (check one) →

1. SHOW ALL WORK. All work must be done in this booklet.
2. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
3. Do all FIVE (5) problems.
4. Exams will be collected 5 minutes before the hour.

Problem	Maximum	Score	Grader
1	16	16	
2	23	23	VR
3	23	23	JDM
4	22	19	TI
5	16	16	K
TOTAL	100	97	

R01	MW 1:00		W. Bertozzi
R02	MW 2:00		W. Bertozzi
R03	MW 3:00		W. Bertozzi
R04	MW 1:00		W. Busza
R05	MW 2:00		W. Busza
R06	MW 3:00		W. Busza
R16	TR 10:00		M. Chen
R17	TR 11:00		M. Chen
R20	TR 2:00		M. Chen
R15	TR 11:00		B. Feng
R24	TR 3:00		B. Feng
R25	TR 10:00		E. Hudson
R26	TR 11:00		E. Hudson
R07	MW 1:00		T. Imai
R08	MW 2:00		T. Imai
R10	TR 9:00		S. Kowalski
R11	TR 10:00		S. Kowalski
R12	TR 11:00		S. Kowalski
R28 (M)	TR 3:00	X	J. McBride
R19	TR 1:00		S. Mtingwa
R21	TR 2:00		S. Mtingwa
R13	TR 9:00		A. Nayeri
R14	TR 10:00		A. Nayeri
R22	TR 1:00		D. Pooley
R23	TR 2:00		D. Pooley
R09	MW 3:00		X. Wen

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Problem 1 (16 points)

Write down the equation for a traveling wave (in the x-z plane) on a string which satisfies the following: The wave propagates in the +z-direction with a speed of 100 m/sec, the amplitude (in the x-direction) is 0.005 m, and the frequency is 400 Hz.

$X = X_0 \sin(kx - \omega t)$

$X_0 = 0.005\,m$

$f = \omega/2\pi = 400\,Hz \Rightarrow \omega = 2\pi \cdot 400\,Hz$

$v = \omega/k = 100\,m/s \Rightarrow k = \frac{\omega}{100\,m/s} = \frac{2\pi \cdot 400\,Hz}{100\,m/s}$

$= 8\pi \cdot 1/m$

$X = 0.005\,m \sin\left(\left(\frac{8\pi}{1m}\right) Z - (2\pi \cdot 400\,Hz)\,t\right)$

Problem 2 (23 points)

A standing electromagnetic plane wave is given by $\vec{E} = 3\hat{z}\cos(\frac{\pi}{2}y)\sin(10^8\pi t)$ V/m. y is in meters, t in seconds, and all angles are in radians.

a. (10 points) What is the wavelength (in m) of the wave, and what is the index of refraction of the medium?

b. (13 points) What is the maximum value (in V/m) that the electric field will have at the location x = 3 m, y = 0.5 m, and z = 2 m?

a. $\lambda = 2\pi/k = \frac{2\pi}{\pi/2} = \boxed{4m}$ ✓

$n = \frac{c}{v} = \frac{ck}{\omega} = \frac{3\times10^8 \cdot \pi/2}{10^8\pi} = 3/2 = \boxed{1.5}$ ✓

b. At this point, $\vec{E} = 3\hat{z}\cos(\pi/2 \cdot 0.5)\sin(10^8\pi t)$

$= \hat{z} \cdot 3 \cdot 1/\sqrt{2} \cdot \sin(10^8\pi t)$ V/m

This value is maximized when the sine term reaches its maximum value of 1, giving a max value of $\boxed{3/\sqrt{2}\ V/m}$ ✓

Problem 3 (23 points)

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A circuit consists of a resistor of 5 Ω, a capacitor of 1 μF, and an ideal self-inductor of 0.01 H. All three are **in series** with a power supply that generates an EMF of $10\sin(\omega t)$ Volt. The internal resistance of the power supply is negligibly small. **The system is at resonance.**

a. (10 points) What is the time averaged power (in Watt) generated by the power supply?

We decrease the frequency of the power supply such that the reactance $(\frac{1}{\omega C} - \omega L)$ becomes 5 Ω; the maximum EMF remains 10 V.

b. (13 points) What now will be the time averaged power generated by the power supply?

a. At resonance, the inductor and capacitor are effectively absent.

$$P = I\mathcal{E} = V^2/R$$

$$\langle P \rangle = \langle V^2/R \rangle = \frac{V_0^2}{2R} = \frac{(10\,V)^2}{2(5\,\Omega)} = \frac{100\,V^2}{10\,\Omega}$$

$$\boxed{= 10\ W}$$

b. Under these conditions, we must use the relation

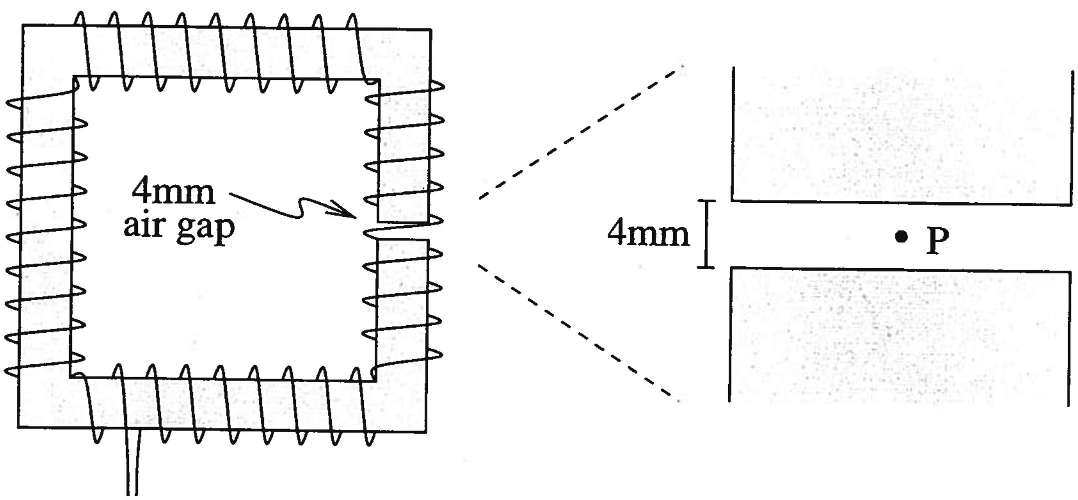
$$\langle P \rangle = \frac{V_0^2}{2Z}\cos\varphi = \frac{V_0^2}{2Z}\cdot\frac{R}{Z} = \frac{V_0^2 R}{2Z^2}$$

$$= \frac{V_0^2 R}{2(R^2 + X^2)} = \frac{(10\,V)^2 \cdot 5\,\Omega}{2((5\,\Omega)^2 + (5\,\Omega)^2)}$$

$$= \frac{500\,V^2\Omega}{2\cdot(25\,\Omega^2 + 25\,\Omega^2)} = \frac{500\,V^2\Omega}{100\,\Omega^2} = 5\,V^2/\Omega = \boxed{5\,W}$$

Problem 4 (22 points)

An iron bar which is 80 cm long is bent in a shape as shown. The ends of the bar do not touch; they are 4 mm apart. There is a coil with 1000 turns (see the figure). The current through the coil is 2 A. $\kappa_M = 200$ for the 80 cm long iron core. What is the approximate magnetic field strength (in Tesla) inside the air gap at P?



Assume the air gap is small enough that there is negligible fringing; the B-field strength must be equal in the air gap as in the iron core in order for Gauss's law for Magnetism to hold. Accordingly,

$$\oint B \cdot dl = \mu_0 \kappa_m I \Rightarrow \oint B/\kappa_m \cdot dl = \mu_0 I$$

$$\Rightarrow \left[\int_{core} \frac{B}{\kappa_{m\,iron}} \cdot dl + \int_{gap} \frac{B}{\kappa_{m\,air}} \cdot dl\right] = \mu_0 N I$$

$$\Rightarrow \left[\frac{B}{200} \cdot 80\text{ cm} + B/1 \cdot 4\text{ mm}\right] = \mu_0 \cdot 1000 \cdot 2A$$

$$\Rightarrow \frac{B \cdot 800mm}{200} + B \cdot 4mm = \mu_0 \cdot 2000A$$

$$\Rightarrow B \cdot 4mm + B \cdot 4mm = \mu_0 \cdot 2000A$$

$$\Rightarrow B \cdot 8mm = \mu_0 \cdot 2000A$$

$$\Rightarrow \boxed{|B| = \frac{\mu_0 \cdot 2000A}{8\,\cancel{mm}}}$$

Problem 5 (16 points)

Here follow 4 "True-False" questions. **For each correct answer you get 4 points, for each incorrect answer, you lose 4 points. You have the option of leaving one or more questions unanswered in which case you will neither gain nor lose points. Your total score for this problem will not be negative.** Please write only a "T" or an "F" in each box, or leave it blank.

a. Red laser light has a smaller wavelength in water than in air.

[T]

f is constant,
v decreases in water.
$\lambda = vT = v/f$ decreases

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b. The time-averaged Poynting vector in a standing electromagnetic wave is zero.

[T]

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c. The following traveling electromagnetic wave is linearly polarized:

$E_x = 0$; $E_y = E_0\, sin(kx + \omega t)$, $E_z = -2E_0\, sin(kx + \omega t)$

[T]

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d. In the case where there is total reflection of light off the surface between two media, the angle of incidence must exceed a certain value which depends on the index of refraction of *both* media.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$\Rightarrow \sin\theta_2 = \sin\theta_1 \cdot \frac{n_1}{n_2}$

[T]